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HYDRONAUTICS, Incorporated

TECHNICAL REPORT 636-1

ROUGH WATER MATING OF ROLL-ON/ROLL-OFF SHIPS WITH BEACH DISCHARGE LIGHTERS

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NOTATION

$a(=a_d), a_s$	Wave amplitude in deep and shallow water respectively
a_1, a_3, b	Coefficients of transformation of Lewis section
A^*, A_s^*	Energy density of the wave spectrum in deep and shallow water respectively
A_{22}, A_{33}, A_{42}	Section added mass for sway, heave and roll respectively
A_z	Ratio between the amplitudes of the heave generated wave to the heaving motion of the ship section
B	Local beam of the ship section
$\overline{BG}$	Vertical distance between the center of buoyancy and the center of gravity
c, c_s	Speed of wave propagation in deep and shallow water respectively
C_s	Sectional coefficient of ship hull
$C_{(subscript)}$	Three-dimensional damping factor for the subscripted motions
d	Water depth
d_h	Distance between the center of gravity of ships
D	Time derivative operator
F_c	Correction factor for the slenderness of the ship

g	Gravitational acceleration
$\overline{GM}$	Metacentric height
$\bar{h}$	Mean of half draft
H	Local draft of the ship section
I_x, I_y, I_z	Moment of inertia of the ship about the X,Y and Z axis respectively
I_{xt}	Total moment of inertia about X-axis including hydrodynamic moment of inertia
$k_{(subscript)}$	Spring constant of the subscripted motion
K_n	Spring constant of the nth cable
K_e, M_e, N_e	Wave exciting moments for rolling, pitching and yawing respectively
$K_{(subscript)}$, $M_{(subscript)}$, $N_{(subscript)}$	Coefficients in the roll, pitch and yaw equation due to the subscripted motions
ℓ_1, ℓ_2	Distance from the stern to the center of gravity of ship
L	Length of ship
m	mass of ship
$\overline{OG}$	Vertical distance from the free surface to the center of gravity of the ship
S	Cross-sectional area of ship hull

t	Time
$T(\beta)$	Response amplitude operator for the particular motion at heading angle β
$T_{(subscript)}$	Period of the subscripted motion
V_w	Wind velocity
W	Total displacement of ship
x, y, z	Complex displacements in surging, swaying and heaving respectively, i.e. $x = x_r + ix_i$, $y = y_r + iy_i$ and $z = z_r + iz_i$.
$\dot{x}, \dot{y}, \dot{z}$	First derivative with respect to time for x,y and z respectively
$\ddot{x}, \ddot{y}, \ddot{z}$	Second derivative with respect to time for x, y and z respectively
X, Y, Z	Perpendicular axes of a rectangular coordinate
X_e, Y_e, Z_e	Wave exciting forces in surging, swaying and heaving respectively
X_R, Y_R, Z_R	Relative displacements between the sterns of two ships in the X,Y, and Z direction respectively
$X_{(subscript)}$, $Y_{(subscript)}$, $Z_{(subscript)}$	Coefficients in the surge, sway and heave equation due to the subscripted motions
Z_{c_b}	Local center of buoyancy of ship section

α_n	Projected angle between the nth cable and the X-axis in the horizontal plane
β	Direction of wave propagation measured from X-axis
γ	Draft-half beam ratio
δ	Phase difference between two ships with respect to the wave at oblique sea
$\epsilon_{(subscript)}$	Phase difference between the subscripted motions and the wave
η	Free surface elevation associated with wave
θ, φ, ψ	Complex rotational displacements in pitching, rolling and yawing respectively, i.e., $\theta = \theta_r + i\theta_i$, $\varphi = \varphi_r + i\varphi_i$, $\psi = \psi_r + i\psi_i$
$\dot{\theta}, \dot{\varphi}, \dot{\psi}$	First derivative with respect to time for θ, φ and ψ respectively
$\ddot{\theta}, \ddot{\varphi}, \ddot{\psi}$	Second derivative with respect to time for θ, φ and ψ respectively
θ_w	Wave direction measured from the direction of the predominant wind
$\lambda (= \lambda_d)$, λ_s	Wave length in deep and shallow water respectively
μ	Coefficient of decay
ξ_s, ξ_b	Position of stern and bow respectively on the dummy axis of ξ
ρ	Density of water
Φ_w, Φ_{w_s}	Wave potential in deep and shallow water respectively
ω	Frequency of the oncoming waves

I. INTRODUCTION

The mating of two large ships in rough seas for the purpose of transferring cargo from one to the other is, as one could expect, an exceptionally difficult maneuver. The basic purpose of developing the beach discharge lighter, USAV LT COL JOHN V. D. PAGE and the roll-on/roll-off ship USNS COMET was to provide such a mating capability. In this way, it was intended to be able to unload the COMET and transfer the materiel to an unimproved beach. Experience has shown, however, that the mating maneuver can not only be difficult but can be dangerous as well (see, for instance, Reference 1). As a result of this study, it was described in Reference 1 that mating could not occur if the relative motion between the two vessels is greater than four feet. Since this condition occurs at relatively low sea states (upper Sea State two) the usefulness of these ships for mating is severely compromised.

In order to investigate ways of improving the ability to mate these two ships at higher sea states than presently possible, an extensive research program embracing both theoretical and experimental studies was undertaken. The major objective of this study was to determine if some heading other than the head seas approach currently used would decrease the relative motion between the two ships. It was felt that some other heading, for instance, beam seas, might prove to be advantages since, in this case, the severe pitch motions should be virtually eliminated. Indeed, one can expect more roll motion in

beam seas, but there are currently available means for controlling these motions (i.e., roll tanks).

In this report, we investigate the ship motions by applying the established linearized, slender-body theory for vessels of arbitrary shape. In the calculation of ship motion, the ship is assumed to be a rigid body with six degrees of freedom. The fluid is assumed to be incompressible, inviscid and irrotational. The perturbation velocities due to the presence of the ship are assumed to be confined to two-dimensional flow between two adjacent control planes, such that slender body theory is applicable. The motions are assumed to be small, hence the equation of motion can be linearized. As a result the motions in the vertical plane are separated from the motions in the horizontal plane. The shape of the ship hull is assumed to be replaceable by the equivalent Lewis form. In evaluation of the wave forces and moments, in addition to the Froude-Kriloff hypothesis, the interference of the ship hull with the water flow in waves was taken into account.

The relative motions between the USNS COMET and the USAV LT COL JOHN V. D. PAGE were investigated for the case of stern to stern mating, as a function of the headings. Motions in irregular seas were investigated by statistical methods using Neumann's wave spectrum for a fully developed sea. This spectrum was modified by means of a cosine-squared distribution for the short-crested sea approximation. Finally, shallow water effects were investigated.

In addition to the above detailed theoretical investigations, a series of experiments were conducted at the Netherlands Ship Model Basin (NSMB). The purpose of these experiments was threefold; first to determine the regular wave transfer functions of each ship in order to check the assumptions made by the linear theory; second, to refine the theory by means of comparison with the data so that the revised theory is a validated and useful tool for further research into ship mating problems; third, to confirm the predictions of the revised theory with regard to the motions and optimum headings by testing the PAGE-COMET system in several realistic, irregular seas.

II. METHOD OF EVALUATION OF SHIP MOTION

(a) Linearized Equations of Motion

A right-hand Cartesian coordinate system, with its origin located at the center of gravity of the ship, was chosen, as shown in Figure 1. The X-axis is positive toward the bow, the Y-axis is positive to port and the Z-axis is positive upward. The linear displacements in the direction of the X, Y and Z axes define the motion of surge x, sway y, and heave z respectively. The angular displacements about the X, Y and Z axes define the motion of roll φ, pitch θ, and sway ψ, respectively. Roll is positive with port upward, pitch is positive with bow downward and yaw is positive with bow to port. The positive direction of forces and moments are similarly defined.

With the assumption of small displacements of rigid body motion, motions in the vertical plane (heave, pitch and surge) are uncoupled with the motions in the horizontal plane (sway, roll and yaw). By balancing the forces due to: (1) inertia of body and fluid, (2) damping, (3) hydrostatic restoring force, (4) mooring force, and (5) the wave exciting force, the following six equations of motion can be written in a general form:

Heave

$$[(m-Z_{\ddot{z}})D^2 - Z_{\dot{z}}D - Z_z]z - (Z_{\ddot{\theta}}D^2 + Z_{\dot{\theta}}D + Z_\theta)\theta = Z_e \qquad [1]$$

Pitch

$$-[M_{\ddot{z}}D^2 + M_{\dot{z}}D + M_z]z + [(I_y - M_{\ddot{\theta}})D^2 - M_{\dot{\theta}}D - M_\theta]\theta - m\,\overline{BG}\,D^2 x = M_e \qquad [2]$$

Surge

$$(mD^2 - X_{\dot{x}}D + k_x)x - m\,\overline{BG}\,D^2\theta = X_e \qquad [3]$$

Sway

$$[(m-Y_{\ddot{y}})D^2 - Y_{\dot{y}}D + k_y]y - (Y_{\ddot{\psi}}D^2 + Y_{\dot{\psi}}D)\psi - (Y_{\ddot{\phi}}D^2 + Y_{\dot{\phi}}D)\phi = Y_e \qquad [4]$$

Yaw

$$-(N_{\ddot{y}}D^2 + N_{\dot{y}}D)y - [(I_z - N_{\ddot{\psi}})D^2 - N_{\dot{\psi}}D + k_\psi]\psi - (N_{\ddot{\phi}}D^2 + N_{\dot{\phi}}D)\phi = N_e \qquad [5]$$

Roll

$$-(K_{\ddot{y}}D^2 + K_{\dot{y}}D)y - (K_{\ddot{\psi}}D^2 + K_{\dot{\psi}}D)\psi + [I_{xt}D^2 - K_{\dot{\phi}}D - K_\phi]\phi = K_e \qquad [6]$$

In Equations [1] to [6], D is the time derivative operator. Several constants about the ship are defined as follows:

m	is the mass of the ship,
$\overline{BG}$	is the vertical distance between the center of buoyancy and the center of gravity, and
I_y and I_z	are the moment of inertia of the ship about the y and z axes respectively.

The remaining coefficients in the left hand side and the wave exciting forces in the right hand side of Equations [1] to [6] are described in the following paragraphs.

1. <u>The Hydrodynamic Coefficients</u>

The hydrodynamic coefficients are exceptionally difficult to estimate accurately. The approach adopted here is to divide the ship into strips and compute these coefficients by applying the existing results derived from slender body theory to each strip along the ship. The overall coefficients are then obtained by integration.

$Z_{\ddot{z}}$ is the coefficient of vertical force due to the added mass in heave, including free-surface effects. Each cross-section is replaced by a Lewis form and the method of Grim (5) is used for calculating the local section force coefficient, A_{33}. The computer program of Grim's method given in the report of Vassilopoulos (6) was used. Thus

$$Z_{\ddot{z}} = -\int_{\xi_s}^{\xi_b} A_{33}\, d\xi \qquad [7]$$

where ξ is the dummy variable along the X-axis and the integration extends from stern ξ_s, to bow ξ_b.

$Z_{\dot{z}}$ is the coefficient of vertical force due to the damping effect of ship sections in heave and can be expressed as

$$Z_{\dot{z}} = - C_z \frac{\rho g^2}{\omega^3} \int_{\xi_s}^{\xi_b} A_z^2 \, d\xi \qquad [8]$$

where

ρ	is the density of water
ω	is the frequency of the oncoming waves
g	is the gravitational acceleration
A_z	is the ratio of the amplitude of the heave generated two-dimensional waves to the amplitude of heaving motion of the ship cross-section. It was obtained from the same source of Grim's work, (3), (4).
C_z	is the three-dimensional damping factor for heave

Z_z is the coefficient of vertical force due to section hydrostatic restoring force in heave and can be expressed as

$$Z_z = - \rho g \int_{\xi_s}^{\xi_b} B \, d\xi \qquad [9]$$

where B is the local beam of the ship section.

The coefficients due to pitching can be easily derived from the local heave coefficients. $Z_{\ddot{\theta}}$ is the coefficient of vertical force due to section added mass in pitching acceleration and can be expressed as

$$Z_{\ddot{\theta}} = \int_{\xi_s}^{\xi_b} A_{33} \, \xi \, d\xi \qquad [10]$$

$Z_{\dot{\theta}}$ is the coefficient of vertical force due to section damping effect in pitching and can be expressed as

$$Z_{\dot{\theta}} = C_\theta \frac{\rho g^2}{\omega^3} \int_{\xi_s}^{\xi_b} A_z^2 \, \xi \, d\xi \qquad [11]$$

where C_θ is the three-dimensional damping factor for pitch.

Z_θ is the coefficient of vertical force due to section hydrostatic restoring force in pitch and can be expressed as

$$Z_\theta = - \rho g \int_{\xi_s}^{\xi_b} B\xi \, d\xi \qquad [12]$$

Since the pitch moment is the vertical force multiplied by the moment arm ξ along the X-axis, the coefficients in Equation [2] are obtained from the following relation

$$M_{\ddot{z}} = \int_{\xi_s}^{\xi_b} A_{33}\, \xi \, d\xi \qquad [13]$$

$M_{\ddot{z}}$ is the coefficient of pitch moment due to section added mass in heaving acceleration.

$$M_{\dot{z}} = C_z \frac{\rho g^2}{\omega^3} \int_{\xi_s}^{\xi_b} A_z^2 \xi \, d\xi \qquad [14]$$

$M_{\dot{z}}$ is the coefficient of pitch moment due to section damping force in heave.

$$M_z = \rho g \int_{\xi_s}^{\xi_b} B\xi \, d\xi \qquad [15]$$

M_z is the coefficient of pitch moment due to section hydrostatic restoring force in heave.

$$M_{\ddot{\theta}} = -\int_{\xi_s}^{\xi_b} A_{33}\, \xi^2 \, d\xi \qquad [16]$$

$M_{\ddot{\theta}}$ is the coefficient of pitch moment due to section added mass in pitching acceleration.

$$M_{\dot{\theta}} = C_\theta \frac{\rho g^2}{\omega^3} \int_{\xi_s}^{\xi_b} A_z^2 \, \xi^2 d\xi \qquad [17]$$

$M_{\dot{\theta}}$ is the coefficient of pitch moment due to section damping force in pitch.

$$M_{\theta} = - \rho g \int_{\xi_s}^{\xi_b} B\xi^2 \, d\xi \qquad [18]$$

M_{θ} is the coefficient of pitch moment due to section hydrostatic restoring force.

In the surge equation of motion, Equation [3], $X_{\dot{x}}$ is the coefficient of longitudinal force due to section damping force. From Reference 4, this coefficient can be expressed approximately as

$$X_{\dot{x}} = \frac{1}{28} Z_{\dot{z}} = - \frac{C_z}{28} \frac{\rho g^2}{\omega^3} \int_{\xi_s}^{\xi_b} A_z^2 \, d\xi \qquad [19]$$

The coefficient of longitudinal force due to a mooring cable, k_x, can be expressed as

$$k_x = \sum_{n=1}^{N} K_n \cos^2 \alpha_n \qquad [20]$$

where N is the number of cables, K_n is the spring constant of the nth cable and α_n is the projected angle between the nth cable and the X-axis.

In Equation [4], $Y_{\ddot{y}}$ is the coefficient of lateral force due to the section added mass in sway acceleration and can be expressed as

$$Y_{\ddot{y}} = -\int_{\xi_s}^{\xi_b} A_{22}\, d\xi \qquad [21]$$

where A_{22} is the section added mass (in sway) and is given by (7)

$$\begin{aligned} A_{22} &= A_{22}' + \frac{2\pi}{\lambda} A_{22}'' \\ &= \frac{\pi}{2} \rho H^2 \left[Q_1 + \frac{8H}{\lambda} (Q_2 + Q_3 - Q_4) \right] \end{aligned} \qquad [22]$$

and

$$Q_1 = \frac{(1 - a_1)^2 + 3a_3^2}{(\gamma b)^2}$$

$$Q_2 = \frac{1}{(\gamma b)^3} (1 - a_1)^2 \left(1 - \frac{a_1}{3} - \frac{3}{5} a_3\right)$$

$$Q_3 = \frac{1}{(\gamma b)^3} a_3^2 \left(\frac{1}{5} - \frac{a_1}{7} - \frac{a_3}{3}\right)$$

$$Q_4 = \frac{2}{(\gamma b)^3} a_3 (1 - a_1) \left(\frac{1}{3} - \frac{a_1}{5} - \frac{3}{7} a_3\right)$$

where λ is the wave length, H is the section draft, γ is the draft-half beam ratio, or $\gamma = \frac{H}{B/2}$ and a_1, a_3, and b are coefficients involved in the transformation of the ship section to its

equivalent Lewis form. From Reference 3, these coefficients can be obtained from the value of γ and the section coefficient $C_s = \frac{S}{B \times H}$, where S is the section area, by the following expression

$$b = 4 \Big/ \left[3(1+\gamma) - \sqrt{9(1+\gamma)^2 - 4\left[2(1+\gamma+\gamma^2)\right] + \frac{8\gamma C_s}{\pi}}\right]$$

$$a_1 = \frac{b}{2}(1 - \gamma) \qquad [23]$$

$$a_3 = \frac{b}{2}(1 + \gamma) - 1$$

$Y_{\dot{y}}$ is the coefficient of lateral force due to the section damping force in sway and can be expressed (4,7) as

$$Y_{\dot{y}} = -\frac{C_y \rho \omega^5}{16 g^2} \int_{\xi_s}^{\xi_b} B^4 \; dy^2 \; d\xi \qquad [24]$$

where $dy = \frac{\pi(1-a_1)}{(1+a_1+a_3)^2}$ and C_y is the three-dimensional damping factor for sway.

k_y is the coefficient of lateral force due to mooring cable and can be expressed as (4)

$$k_y = \sum_1^N K_n \sin^2 \alpha_n \qquad [25]$$

$Y_{\ddot{\phi}}$ is the coefficient of lateral force due to section added mass in rolling acceleration and can be expressed as

$$Y_{\ddot{\phi}} = -\int_{\xi_s}^{\xi_b} \left[A_{42} + \overline{OG}\, A_{22} \right] d\xi \qquad [26]$$

where $\overline{OG}$ is the vertical distance from the free surface to the center of gravity of the ship. A_{42} is the added mass in roll due to sway motion and is given by Hu (7) as

$$A_{42} = A_{42}{}' + \frac{2\pi}{\lambda} A_{42}{}''$$

$$= \frac{\pi}{2} \rho H^3 \left[P_1 + \frac{4H}{\lambda} (P_2 + P_3) \right] \qquad [27]$$

and

$$P_1 = \frac{-8}{\frac{\pi}{2}(\gamma b)^3} \left[\frac{1}{3} a_1 (1-a_1) + \frac{1}{15} a_3 (4+4a_1 - 5a_1^2) - \frac{1}{35} a_3^2 (20-7a_1) \right]$$

$$P_2 = -\frac{1}{\pi(\gamma b)^4} (a_1 + a_1 a_3 - 4a_3) \times$$

$$\left\{ \pi^2 (1-a_1) - \left(8 - \frac{\pi}{2}\right) [a_3 + a_1(1-a_1)] + \frac{16}{9} a_3 (4a_1 - 3) + \frac{56}{15} a_3^2 \right\}$$

$$P_3 = -\frac{a_3}{\pi(\gamma b)^4}\left\{5\pi^2(1-a_1) - 2\left(\frac{160}{9}-\pi^2\right)[a_3+a_1(1-a_1)] + \left(\frac{128}{9}-\frac{\pi^2}{2}\right)a_3(4a_1-3) + \frac{10304}{525}a_3^2\right\}$$

where a_1, a_3, and b are given in Equation [23].

$Y_{\dot{\phi}}$ is the coefficient of lateral force due to section damping force in rolling and can be expressed as

$$Y_{\dot{\phi}} = -\overline{BG}\ Y_{\dot{y}} \qquad [28]$$

$Y_{\ddot{\psi}}$ is the coefficient of lateral force due to section added mass in yawing acceleration and can be expressed as

$$Y_{\ddot{\psi}} = -\int A_{22}\,\xi\,d\xi \qquad [29]$$

$Y_{\dot{\psi}}$ is the coefficient of lateral force due to section damping force in yawing and can be expressed as

$$Y_{\dot{\psi}} = -\frac{C_{\psi}\rho\omega^5}{16g^2}\int_{\xi_s}^{\xi_b} B^4(dy)^2\,\xi\,d\xi \qquad [30]$$

where C_{ψ} is the three-dimensional damping factor for yaw.

The coefficients in the yaw equation, or Equation [5] are obtained immediately from those of sway in the following:

$$N_{\ddot{y}} = -\int_{\xi_s}^{\xi_b} A_{22}\, \xi \, d\xi \qquad [31]$$

$N_{\ddot{y}}$ is the coefficient of yaw moment due to section added mass in sway acceleration.

$$N_{\dot{y}} = - C_y \frac{\rho\omega^5}{16g^2} \int_{\xi_s}^{\xi_b} B^4 (dy)^2 \, \xi \, d\xi \qquad [32]$$

$N_{\dot{y}}$ is the coefficient of yaw moment due to section damping force in sway.

$$N_{\ddot{\phi}} = -\int_{\xi_s}^{\xi_b} [A_{42} + (\overline{OG})\, A_{22}]\, \xi \, d\xi \qquad [33]$$

$N_{\ddot{\phi}}$ is the coefficient of yaw moment due to section added mass in rolling acceleration.

$$N_{\dot{\phi}} = - |BG| \frac{\rho\omega^5}{16g^2} \int_{\xi_s}^{\xi_b} B^4 (dy)^2 \, \xi \, d\xi \qquad [34]$$

$N_{\dot{\phi}}$ is the coefficient of yaw moment due to section damping force in roll.

$$N_{\ddot{\psi}} = -\int_{\xi_s}^{\xi_b} A_{22}\xi^2\,d\xi \qquad [35]$$

$N_{\ddot{\psi}}$ is the coefficient of yaw moment due to section added mass in yaw acceleration.

$$N_{\dot{\psi}} = -C_{\psi}\frac{\rho\omega^5}{16g^3}\int_{\xi_s}^{\xi_b} B^4\,(dy)^2\,\xi^2\,d\xi \qquad [36]$$

$N_{\dot{\psi}}$ is the coefficient of yaw moment due to section damping force in yaw. k_{ψ} is the coefficient of yaw moment due to mooring cables and can be expressed as

$$k_{\psi} = \frac{L^2}{4}\sum_{1}^{N} K_n \sin^2 \alpha_n \qquad [37]$$

where L is the total length of the ship.

In Equation [6], $K_{\ddot{y}}$ is the coefficient of roll moment due to section added mass in yaw acceleration and can be expressed as

$$K_{\ddot{y}} = -\int_{\xi_s}^{\xi_b}\left[A_{42} + \overline{OG}\,A_{22}\right]d\xi \qquad [38]$$

$K_{\dot{y}}$ is the coefficient of roll moment due to section damping force in sway and can be expressed as

$$K_{\dot{y}} = - \overline{BG} \frac{\rho \omega^2}{16 g^2} \int_{\xi_s}^{\xi_b} B^4 (dy)^2 \, d\xi \qquad [39]$$

I_{xt} is the total moment of inertia of roll, including added mass. The value of I_{xt} can be found through model tests. In this report, this value is assumed to be given implicitly by the roll period of the ship.

$K_{\dot{\phi}}$ is the coefficient of roll moment due to section damping force in roll. This value is approximately varied between 0.05 to 0.10 of the critical roll damping of the ship. Thus,

$$K_{\dot{\phi}} = - \mu \left[2(I_{xt}) \frac{2\pi}{T_\varphi} \right] \qquad [40]$$

where T_φ is the roll period of the ship and μ is the coefficient of decay. Two values of μ of 0.05 and 0.075 were used for the computation in this report.

K_φ is the coefficient of roll moment due to hydrostatic restoring force in roll and can be expressed as

$$K_\varphi = - W(\overline{GM}) \qquad [41]$$

where W is the total displacement of the ship and $\overline{GM}$ is the metacentric height.

$K_{\ddot{\psi}}$ is the coefficient of roll moment due to section added mass in yaw acceleration and can be expressed as

$$K_{\ddot{\psi}} = -\int_{\xi_s}^{\xi_b} \left[A_{42} + \overline{OG}\, A_{22}\right] \xi \; d\xi \qquad [42]$$

$K_{\dot{\psi}}$ is the coefficient of roll moment due to section damping force in yaw and can be expressed as

$$K_{\dot{\psi}} = - \overline{BG}\, \frac{\rho\omega^3}{16g^2} \int_{\xi_s}^{\xi_b} B^4 (dy)^2 \; \xi \; d\xi \qquad [43]$$

For each ship, the above equations are used to compute each of the coefficients in the left-hand sides of Equations [1] - [6]. The following section describes the determination of the wave exciting forces which appear on the right-hand sides of these equations.

ii. The Wave Exciting Forces and Moments

The wave exciting forces and moments are expressed for a unit amplitude wave based upon potential theory and slender body theory (4). The waves are assumed to be propagating in a direction defined by the angle β, where β is defined as the angle between the X-axis and the normal to the wave crests. β is lying in the range $-\pi < \beta < \pi$. The wave propagation speed c is always taken positive along the radial line defining the wave propagation direction, as shown in Figure 2.

The complex wave potential Φ for a unit amplitude wave in deep water, referred to axes on the free surface, is given by

$$\Phi_w = c\, e^{\frac{2\pi}{\lambda}[z - i(x \cos \beta + y \sin \beta]}\, e^{i\omega t} \qquad [44]$$

and the free surface elevation associated with it is

$$\eta = \mathrm{Re}\left\{\frac{1}{g}\frac{\partial \Phi_w}{\partial t}\bigg|_{z=0}\right\} = \frac{c\omega}{g} \sin\left[\frac{2\pi}{\lambda}(x \cos \beta + y \sin \beta) - ct\right] \qquad [45]$$

where Re represents real part of { }, ω is the frequency of waves, or $\omega = \frac{2\pi c}{\lambda}$. For evaluating the wave exciting forces and moments, the following simplifications are made: (1) due to the characteristic exponential decay of waves, the orbital velocities are evaluated at a mean half-draft $\bar{h} = H/2$, that is $z = -\bar{h}$; (2) the forces and moments are first evaluated on the x-z plane, that is $y = 0$, and an approximate correction factor which accounts for the influence of slenderness of the ship later applied. Thus, by considering the hydrostatic pressure, the inertial contribution, the effect of damping due to the relative motion between body velocities and the wave orbital velocities, and the lateral orbital velocity gradient, the wave exciting forces and moments can be expressed as follows:

Wave exciting force for heave

$$Z_e = F_c \, \mathrm{Re} \left\{ i \rho g \left[\int_{\xi_s}^{\xi_b} B \, e^{-i \frac{2\pi\xi}{\lambda} \cos \beta} d\xi \right] e^{i\omega t} \right.$$

$$- i \rho \omega^2 e^{-\frac{2\pi \bar{h}}{\lambda}} \left[\int_{\xi_s}^{\xi_b} S + \frac{A_{33}}{\rho} \, e^{-i \frac{2\pi\xi}{\lambda} \cos \beta} d\xi \right] e^{i\omega t}$$

$$\left. - \omega \cdot e^{-\frac{2\pi \bar{h}}{\lambda}} \left[\int_{\xi_s}^{\xi_b} \frac{\rho g^2}{\omega^3} A_z^2 \, e^{-i \frac{2\pi\xi}{\lambda} \cos \beta} d\xi \right] e^{i\omega t} \right\} \qquad [46]$$

where the first term is due to the buoyancy alternations as the wave passes the ship hull, the second term is due to the inertia and the third term is due to the damping. S is defined as the ship sectional area. F_c is a correction factor which accounts approximately for the influence of the slenderness of the ship in terms of the ratio of beam and wave length as

$$F_c = \frac{\sin\left(\frac{\pi B}{\lambda} \sin \beta\right)}{\frac{\pi B}{\lambda} \sin \beta} \qquad [47]$$

All other symbols are as defined earlier.

The wave exciting moment for pitch is

$$M_e = F_c \cdot \mathrm{Re}\left\{ -i\rho g \left[\int_{\xi_s}^{\xi_b} B\, e^{-i\left(\frac{2\pi\xi}{\lambda}\right)\cos\beta} \xi\, d\xi \right] e^{i\omega t} \right.$$

$$+ i\rho\omega^2 \left[\int_{\xi_s}^{\xi_b} \left(S + \frac{A_{33}}{\rho}\right) e^{-i\left(\frac{2\pi\xi}{\lambda}\right)\cos\beta} \xi\, d\xi \right] e^{i\omega t}$$

$$\left. + \omega \cdot e^{-\frac{2\pi\bar{h}}{\lambda}} \left[\int_{\xi_s}^{\xi_b} \frac{\rho g^2 A_z^2}{\omega^3} e^{i\left(\frac{2\pi\xi}{\lambda}\cos\beta\right)} \xi\, d\xi \right] e^{i\omega t} \right\} \qquad [48]$$

The wave exciting force for surge is

$$X_e = F_c \cdot \mathrm{Re}\left\{ -\rho\omega^2 e^{-\frac{2\pi\bar{h}}{\lambda}} \cos\beta \left[\int_{\xi_s}^{\xi_b} S\, e^{-i\left(\frac{2\pi\xi}{\lambda}\cos\beta\right)} d\xi \right] e^{i\omega t} \right\}$$

[49]

where the exciting forces due to damping and orbital velocity gradient are neglected.

The exciting force for sway is

$$Y_e = F_c \cdot Re \left\{ - \rho \omega^2 \; e^{-\frac{2\pi \bar{h}}{\lambda}} \sin \beta \left[\int_{\xi_s}^{\xi_b} \left(S + \frac{A_{22}}{\rho} \right) e^{-i \frac{2\pi\xi}{\lambda} \cos\beta} d\xi \right] e^{i\omega t} \right.$$

$$+ \omega \; e^{-\frac{2\pi\bar{h}}{\lambda}} \sin \beta \left[\int_{\xi_s}^{\xi_b} \frac{\rho\omega^6 B^4}{16 g^2} (dy)^2 \; e^{-i \frac{2\pi\xi}{\lambda} \cos \beta} d\xi \right] e^{i\omega t}$$

$$\left. - \rho \frac{2\pi}{\lambda} \omega^2 \; e^{-\frac{2\pi\bar{h}}{\lambda}} \sin \beta \left[\int_{\xi_s}^{\xi_b} \frac{A_{42}'}{\rho} \; e^{-i \frac{2\pi\xi}{\lambda} \cos \beta} d\xi \right] e^{i\omega t} \right\} \quad [50]$$

where the first term is due to the inertia, the second term is due to the damping and the third term is due to the lateral velocity gradient.

The wave exciting moment for yaw is

$$N_e = F_c \cdot \mathrm{Re}\left\{-\rho\omega^2 e^{-\frac{2\pi\bar{h}}{\lambda}} \sin\beta \left[\int_{\xi_s}^{\xi_b} \left(S + \frac{A_{22}}{\rho}\right) e^{-i\frac{2\pi\xi}{\lambda}\cos\beta} \xi\, d\xi\right] e^{i\omega t}\right.$$

$$+ \omega e^{-\frac{2\pi\bar{h}}{\lambda}} \sin\beta \left[\int_{\xi_s}^{\xi_b} \frac{\rho\omega^2 B^4}{16 g^2} (dy)^2 e^{-i\frac{2\pi\xi}{\lambda}\cos\beta} \xi\, d\xi\right] e^{i\omega t}$$

$$\left. - \frac{2\pi}{\lambda}\omega^2 e^{-\frac{2\pi\bar{h}}{\lambda}} \sin\beta \left[\int_{\xi_s}^{\xi_b} \frac{A_{42}'}{\rho} e^{-i\frac{2\pi\xi}{\lambda}\cos\beta} \xi\, d\xi\right] e^{i\omega t}\right\} \qquad [51]$$

The wave exciting moment for roll is

$$K_e = F_c \cdot \mathrm{Re}\left\{-\rho\omega^2 e^{-\frac{2\pi\bar{h}}{\lambda}} \sin\beta \left[\int_{\xi_s}^{\xi_b} \left(\frac{A_{42}}{\rho} - SZ_{cb} - \frac{B^3}{12}\right) e^{-i\frac{2\pi\xi}{\lambda}\cos\beta} d\xi\right] e^{i\omega t}\right.$$

$$\left. - \frac{2\pi\omega^2}{\lambda} e^{-\frac{2\pi\bar{h}}{\lambda}} \sin\beta \left[\int_{\xi_s}^{\xi_b} A_{44}\, e^{-i\frac{2\pi\xi}{\lambda}\cos\beta} d\xi\right] e^{i\omega t}\right\}$$

$$- \overline{OG} \cdot Y_e \qquad [52]$$

where Z_{cb} is the local center of buoyancy of ship section, L is the ship length and $\overline{OG}$ is the vertical distance from the free surface to the center of gravity of the ship. In Equation [52], A_{44} is the added mass in roll (8) and can be expressed for a Lewis form section as

$$A_{44} = \frac{\varrho}{\pi} \int_{\xi_s}^{\xi_b} \left(\frac{B}{b}\right)^4 \left[a_1^2 (1+a_3)^2 + \frac{8}{9} a_1 a_3 (1+a_3) + \frac{16}{9} a_3^2\right] d\xi \qquad [53]$$

where a_1, a_3 and b are given in Equation [23].

(b) Solution of the Equations of Motion

Let the complex harmonic solution of the equations of motion be of the following form:

$$\left.\begin{aligned} x &= (x_r + ix_i)\, e^{i\omega t} \\ y &= (y_r + iy_i)\, e^{i\omega t} \\ z &= (z_r + iz_i)\, e^{i\omega t} \\ \theta &= (\theta_r + i\theta_i)\, e^{i\omega t} \\ \varphi &= (\varphi_r + i\varphi_i)\, e^{i\omega t} \\ \psi &= (\psi_r + i\psi_i)\, e^{i\omega t} \end{aligned}\right\} \qquad [54]$$

Substituting Equation [54] into Equations [1] - [6], we obtain six simultaneous, complex equations for the twelve unknowns (the real and imaginary parts of the above motions). These equations can be reduced to a pair of real equations by separating the real and imaginary parts of the complex equations. For the longitudinal motions,

$$[A]\left\{z_r,\ z_i,\ \theta_r,\ \theta_i,\ x_r,\ x_i\right\} = \left\{Z_{er},\ Z_{ei},\ \theta_{er},\ \theta_{ei},\ X_{er},\ X_{ei}\right\} \qquad [55]$$

$$[B]\left\{y_r,\ y_i,\ \psi_r,\ \psi_i,\ \varphi_r,\ \varphi_i\right\} = \left\{Y_{er},\ Y_{ei},\ N_{er},\ N_{ei},\ \varphi_{er},\ \varphi_{ei}\right\} \qquad [56]$$

where

$$[A] = \begin{bmatrix} -(m-Z_{\ddot z})\omega^2 - Z_z + k_z & Z_{\dot z}\omega & Z_{\ddot\theta}\omega^2 - Z_\theta & Z_{\dot\theta}\omega & 0 & 0 \\ -Z_{\dot z}\omega & -(m-Z_{\ddot z})\omega^2 - Z_z + k_z & -Z_{\dot\theta}\omega & Z_{\ddot\theta}\omega^2 - Z_\theta & 0 & 0 \\ M_{\ddot z}\omega^2 - M_z & M_{\dot z}\omega & -(I_y - M_{\ddot\theta})\omega^2 - M_\theta & M_{\dot\theta}\omega & m\,\overline{BG}\,\omega^2 & 0 \\ -M_{\dot z}\omega & M_{\ddot z}\omega^2 - M_z & -M_{\dot\theta}\omega & -(I_y - M_{\ddot\theta})\omega^2 - M_\theta & 0 & 0 \\ 0 & 0 & m\,\overline{BG}\,\omega^2 & 0 & -m\omega^2 + k_x & X_{\dot x}\omega \\ 0 & 0 & 0 & m\,\overline{BG}\,\omega^2 & -X_{\dot x}\omega & -m\omega^2 + k_x \end{bmatrix} \qquad [55a]$$

and

$$[B] = \begin{bmatrix} -(m-Y_{\ddot{y}})\omega^2+k_y & Y_{\dot{y}}\omega & Y_{\ddot{\psi}}\omega^2 & Y_{\dot{\psi}}\omega & Y_{\ddot{\phi}}\omega^2 & Y_{\dot{\phi}}\omega \\ Y_{\dot{y}}\omega & -(m-Y_{\ddot{y}})\omega^2+k_y & -Y_{\dot{\psi}}\omega & Y_{\ddot{\psi}}\omega^2 & -Y_{\dot{\phi}}\omega & Y_{\ddot{\phi}}\omega^2 \\ N_{\ddot{y}}\omega^2 & N_{\dot{y}}\omega & -(I_z-N_{\ddot{\psi}})\omega^2+k_{\psi} & N_{\dot{\psi}}\omega & N_{\ddot{\phi}}\omega^2 & N_{\dot{\phi}}\omega \\ -N_{\dot{y}}\omega & N_{\ddot{y}}\omega^2 & -N_{\dot{\psi}}\omega & -(I_z-N_{\ddot{\psi}})\omega^2+k_{\psi} & -N_{\dot{\phi}}\omega & N_{\ddot{\phi}}\omega^2 \\ K_{\ddot{y}}\omega^2 & K_{\dot{y}}\omega & K_{\ddot{\psi}}\omega^2 & K_{\dot{\psi}}\omega & -I_{xt}\omega^2-k_{\phi} & K_{\dot{\phi}}\omega \\ -K_{\dot{y}}\omega & K_{\ddot{y}}\omega^2 & -K_{\dot{\psi}}\omega & K_{\ddot{\psi}}\omega^2 & -K_{\dot{\phi}}\omega & -I_{xt}\omega^2-K_{\phi} \end{bmatrix} \quad [56a]$$

Therefore, the amplitude and the phase with respect to the phase of wave at the center of gravity of the ship are given as:

$$
\begin{aligned}
x &= (x_r^2 + x_i^2)^{\frac{1}{2}} & \epsilon_x &= \tan^{-1} \frac{x_i}{x_r} \\
y &= (y_r^2 + y_i^2)^{\frac{1}{2}} & \epsilon_y &= \tan^{-1} \frac{y_i}{y_r} \\
z &= (z_r^2 + z_i^2)^{\frac{1}{2}} & \epsilon_z &= \tan^{-1} \frac{z_i}{z_r} \\
\theta &= (\theta_r^2 + \theta_i^2)^{\frac{1}{2}} & \epsilon_\theta &= \tan^{-1} \frac{\theta_i}{\theta_r} \\
\varphi &= (\varphi_r^2 + \varphi_i^2)^{\frac{1}{2}} & \epsilon_\varphi &= \tan^{-1} \frac{\varphi_i}{\varphi_r} \\
\psi &= (\psi_r^2 + \psi_i^2)^{\frac{1}{2}} & \epsilon_\psi &= \tan^{-1} \frac{\psi_i}{\psi_r}
\end{aligned}
\qquad [57]
$$

With the amplitude and phase of the motions known, we are able to calculate the relative motion of two ships in regular seas.

(c) Relative Motions Between Two Ships

The relative motions between two ships in regular waves can be obtained immediately from the six motions of each individual ship. We are interested in the relative motions at the junction of the ships. Three relative translations, X_R, Y_R and Z_R and the relative rotation φ_R will be calculated.

As shown in Figure 3, the ships were oriented stern to stern. The distance between the center of gravity of ships is d_h. With the coordinate axes located on the center of gravity of ship A, the motions of ship B have a phase difference δ,

$$\delta = \frac{2\pi d_h \cos \beta}{\lambda} \qquad [58]$$

which leads the motions of ship A.

Let the motions of ship A be subscripted by 1 and the motions of ship B be subscripted by 2. The relative motions can be expressed as follows:

$$X_R = \left\{ \left[x_1 \cos \epsilon_{x_1} - x_2 \cos \left(\epsilon_{x_2} + \delta \right) \right]^2 + \left[x_2 \sin \left(\epsilon_{x_2} + \delta \right) - x_1 \sin \epsilon_{x_1} \right]^2 \right\}^{\frac{1}{2}} \qquad [59]$$

$$\begin{aligned} Y_R = \Big\{ \Big[& y_1 \cos \epsilon_{y_1} - \ell_1 \psi_1 \cos \epsilon_{\psi_1} \\ & - y_2 \cos \left(\epsilon_{y_2} + \delta \right) - \ell_2 \psi_2 \cos \left(\epsilon_{\psi_2} + \delta \right) \Big]^2 \\ & + \Big[- y_1 \sin \epsilon_{y_1} + \ell_1 \psi_1 \sin \epsilon_{\psi_1} \\ & + y_2 \sin \left(\epsilon_{y_2} + \delta \right) + \ell_2 \psi_2 \sin \left(\epsilon_{\psi_2} + \delta \right) \Big]^2 \Big\}^{\frac{1}{2}} \end{aligned} \qquad [60]$$

$$Z_R = \Big\{ \Big[z_1 \cos \epsilon_{z_1} + \ell_1 \theta_1 \cos \epsilon_{\theta_1}$$

$$- z_2 \cos\left(\epsilon_{z_2} + \delta\right) + \ell_2 \theta_2 \cos\left(\epsilon_{\theta_2} + \delta\right)\Big]^2$$

$$+ \Big[- z_1 \sin \epsilon_{z_1} - \ell_1 \theta_1 \sin \epsilon_{\theta_1}$$

$$+ z_2 \sin\left(\epsilon_{z_2} + \delta\right) - \ell_2 \theta_2 \sin\left(\epsilon_{\theta_2} + \delta\right)\Big]^2 \Big\}^{\frac{1}{2}} \qquad [61]$$

$$\varphi_R = \Big\{ \Big[\varphi_1 \cos \epsilon_{\varphi_1} - \varphi_2 \cos\left(\epsilon_{\varphi_2} + \delta\right)\Big]^2$$

$$+ \Big[\varphi_2 \sin\left(\epsilon_{\varphi_2} + \delta\right) - \varphi_1 \sin \epsilon_{\varphi_1}\Big]^2 \Big\}^{\frac{1}{2}} \qquad [62]$$

where ℓ_1 and ℓ_2 are the distances from stern to the center of gravity of the ship for ship A and B respectively.

(d) Motions in Irregular Sea

Motions and relative motions in irregular sea were computed by statistical methods using the Neumann wave spectrum, i.e., the spectral energy density of the wave for a unidirectional fully developed sea, $A^2(\omega)$, (Figure 2), can be represented by

$$A^2(\omega) = C\,\omega^{-6}\, e^{-2g^2/(\omega V_w)^2} \qquad [63]$$

where

C = 51.5 ft^2/sec^5 and is an empirical constant

V_w = wind velocity in ft/sec

ω = wave frequency

g = gravitational acceleration

The spectral density of any particular motion is then given by

$$\Phi(\omega,\beta) = |T(\beta)|^2 A^2(\omega) \qquad [64]$$

where

$T(\omega,\beta)$ = response amplitude operator for the particular motion at heading angle β.

The root mean square value σ of the motion can be obtained by

$$\sigma(\beta) = \left\{ \frac{1}{2} \int_0^\infty |T(\omega,\beta)|^2 A^2(\omega)\, d\omega \right\}^{\frac{1}{2}} \qquad [65]$$

For a non-unidirectional sea the waves are considered coming from all directions from $\theta_w = -\pi/2$ to $\pi/2$ with respect to the direction of predominant wind and the spectral density of wave is represented by

$$A^2(\omega,\theta) = \frac{2}{\pi} c\, \omega^{-6}\, e^{-2g^2/(\omega V_w)^2} \cos^2\theta_w, \quad \text{for } -\pi/2 < \theta_w < \pi/2$$

$$= 0 \qquad \text{otherwise} \qquad [66]$$

where θ_w is measured from the direction of predominant wind (see Figure 3), and the motion spectrum is given by

$$\Phi(\omega,\beta) = \int_{-\pi/2}^{\pi/2} |T(\omega,\beta)|^2 A^2(\omega,\theta_w)d\theta_w \qquad [67]$$

The root mean square value is also given by Equation [65].

The wind speed for each sea state is given as

Sea State	V_w in knots
3	14
4	17
5	22

III. THEORETICAL RESULTS

Computations were carried out for ship motions of the COMET and the PAGE at zero speed in deep water by the linearized equations of motion based on the strip method, as described previously. The principal characteristics of these two ships are listed in Table 1. The results are presented in the following three parts:

(a) Ship Responses due to Unit-Amplitude, Regular Waves

The ship responses and phase difference between the motion and the wave amplitude for surge, heave, pitch, sway, roll and yaw due to unit-amplitude, regular waves are shown in

Figure 4 for the COMET and in Figure 5 for the PAGE as a function of the wavelength and the headings. Only three headings are shown for each ship. In Figures 4 and 5 the dashed curves represent the case where a lower value of $K_{\phi} = 0.1\ I_{xt}\omega_s$ was used and the three-dimensional damping factors were not considered. The solid curves represent the case where $K_{\phi} = 0.15\ I_{xt}\omega_s$ and the three-dimensional damping factors were employed.

(b) Ship Responses in Unidirectional and Non-Unidirectional Seas

Figures 6 and 7 show the root-mean-square value of surge, heave, pitch, sway, roll and yaw in unidirectional seas for Sea States 3, 4 and 5 for the COMET and the PAGE, respectively. The description of a unidirectional sea is given in Figure 2. Computations were made for intervals of 5 degrees in the heading angle β. Similarly, Figures 8 and 9 show the root-mean-square values of surge, heave, pitch, sway, roll and yaw in non-unidirectional seas for Sea States 3, 4 and 5 for the COMET and the PAGE respectively. The description of a non-unidirectional sea is given in Figure 3. Both the angle β and the angle θ were taken at intervals of 5 degrees in the computation.

(c) Relative Motions in Unidirectional and Non-Unidirectional Seas

Figures 10 and 11 show the root-mean-square values of the relative displacement X_R, Y_R and Z_R and the relative rotation φ_R at the junction between the COMET and the PAGE in a stern to stern mating at Sea States 3, 4 and 5, as a function of

the headings in unidirectional and non-unidirectional seas respectively. The distance between the sterns of two ships was kept at 10 feet apart in Figures 10 and 11. However, additional computation shows that a variation in the distance from 5 to 15 feet does not change the results significantly.

As shown in Figure 11, the results for a non-unidirectional sea, which is considered to be closer to a real sea, show no absolute optimum headings. In Sea State 5, a beam sea is preferred for relative heave and surge, but results in relatively large values of sway and roll. For Sea States 3 and 4, it seems that head seas and following seas are slightly preferred. However, in view of the fact that very large individual ship motions are introduced in beam sea for Sea State 5, as shown in Figures 8 and 9, it is still best to choose head seas and following seas.

IV. SHALLOW WATER EFFECT IN SHIP MOTION

The need to investigate ship motions in shallow water arises for two reasons: (1) the prototype mating may possibly be performed in relatively shallow water, and (2) because of the limitation of the test basin at the Netherlands Ship Model Basin, the experiments of ship motion could only be performed at a relatively shallow water depth corresponding to a depth of 100 feet. However, no theory has been developed so far for the evaluation of the hydrodynamic coefficients of ship sections to include the effect of shallow water. Accordingly, simplifications were then made in considering ship motions in shallow water, as follows:

(1) All added mass coefficients for ship sections are assumed to remain unchanged from the deep water values.

(2) The wave period is assumed to be constant as the waves advance from deep water to shallow water.

(3) No reflection of energy takes place as the depth changes.

From assumptions (2) and (3), Burnside (9) derived the relation of amplitude and progressive speed of waves in deep water to that in shallow water as follows:

$$\frac{c_s}{c_d} = \frac{\lambda_s}{\lambda_d} = \tanh \frac{2\pi d}{\lambda_s} \qquad [68]$$

$$\frac{a_s^2}{a_d^2} = \frac{2 \cosh^2\left(\frac{2\pi d}{\lambda_s}\right)}{\frac{4\pi d}{\lambda_s} + \sinh\left(\frac{4\pi d}{\lambda_s}\right)} \qquad [69]$$

where d is the depth of water and the subscripts d and s denote the parameters in deep water and shallow water respectively. Furthermore, the wave potential as given in Equation [45] should be replaced by

$$\phi_{w_s} = c_s \frac{\cosh \frac{2\pi}{\lambda_s}(z+d)}{\cosh \frac{2\pi d}{\lambda_s}} e^{-\frac{2\pi}{\lambda_s}(x \cos\beta + y \sin\beta)} e^{i\omega t} \quad [70]$$

for the shallow water wave potential.

By assumption (1), the ship motion in regular waves in shallow water can be computed by introducing Equation [68] and [70]. The wave spectral energy density as given in Equation [63] for deep water can then be modified by Equation [69] as an approximation for wave spectral energy density in shallow water, that is

$$A_s^2(\omega) = 51.5 \left[\frac{2 \cosh^2\left(\frac{2\pi d}{\lambda_s}\right)}{\frac{4\pi d}{\lambda_s} + \sinh\left(\frac{4\pi d}{\lambda_s}\right)} \right] \omega^{-6} \; e^{-2g^2/(\omega V_w)^2} \quad [71]$$

Equation [71] is used for the computation of ship motion in irregular waves in shallow water.

The change in wave length and the change in the Neumann's spectral energy density due to the shallow water effect are shown in Figures 12 and 13 respectively. Equations [68] and [69] were also plotted in Figures 12 and 13 respectively.

Computations were carried out for ship relative motions in surge, heave, sway and roll at water depths of 200, 100 and 50 feet in non-unidirectional seas. The results are shown in Figure 14.

It is seen that the ~~effect of~~ shallow water is to increase the relative motion in sway and roll, but to reduce the relative motion in heave. For relative surge, the effect of shallow water is small.

V. COMPARISON BETWEEN THEORY AND EXPERIMENT OF SHIP MOTIONS AT WATER DEPTH OF 100 FEET AND DISCUSSION

(a) Test in Regular Waves

Model tests of the responses of the COMET and the PAGE in regular waves were carried out at the Netherlands Ship Model Basin at a depth equivalent to 100 feet (12). The models were built to a 1:30 scale and were oriented stern to stern in the basin. Measurements were taken at the same time for both ships assuming that the interference effects are small. Three different headings were tested, i.e., head seas, beam quartering seas and beam seas for the COMET and following seas, bow quartering seas and beam seas for the PAGE. The original test results are included in Appendix A. These results were recalculated to give the responses due to unit amplitude waves. The recalculated results are shown in Appendix B.

A plot of results obtained from theory and experiment are shown in Figures 15 and 16 for the COMET and PAGE respectively. In the theoretical results, as computed by the method described previously for a water depth of 100 feet, the roll damping coefficient has been calibrated against the experimental data to obtain consistent values of maximum roll motion. The value of roll damping thus found is

$$K_{\phi} = 0.15\ I_{xt}\omega$$

for both ships.

In addition, it was found advantageous to adjust other damping coefficients in the equations. Here, we used the three dimensional damping factors for heave, pitch, sway and yaw given by Havelock (10) and Hu (11) which are tabulated in Table 2. In Figures 15 and 16, the solid curves represent the case where the three dimensional damping factors are used, and the dashed curves represent the case of neglecting the three dimensional effect on the damping factor. It is seen that, in most cases, the three dimensional damping factors do bring the agreement between experiment and theory closer.

In general, the agreement between theory and experiment on ship motion in regular waves are satisfactory. The degree of agreement for all the cases tested are graded as shown in Table 3. The very strong coupling effect between the sway motion due to roll, as indicated in the theory for the PAGE around its resonant frequency in beam seas does not appear in the experiment. This is the worst case among all the results obtained.

(b) Test in Irregular Waves

Model tests of the relative motions between the sterns of COMET and PAGE were carried out in the same basin and the same water depth (100 feet) as in regular wave tests described previously for Sea States 3, 4 and 5. Two headings, head and

following sea, were test[illegible] The mooring lines for the COMET and the experimental arrangements for the irregular wave test are shown in Figures 17 and 18 for head sea and following sea respectively. Two different weights of the mooring lines were used, i.e., 18 and 55 pounds per foot, in the test. The measured spectra of the generated waves and the relative motions between COMET and PAGE are shown in Figures 19 to 22. The R.M.S.* values of the relative motion obtained from the tests are tabulated in Table 4 and are also shown in Figure 23 in comparison to theory. It should be noted that the difference in the unit weight of the mooring lines, and therefore its stiffness as mentioned above, only makes negligible change in theoretical values of the motion. However, the experimental results show some difference.

As shown in Figure 23, the measured R.M.S. values of the relative motion is much higher than the theory predicted for relative surge. For relative sway and roll, the theory predicts no motion in head and following sea since the wave is unidirectional and the ships are symmetric about their longitudinal centerplane. However, it is seen from Figure 23 that appreciable relative motion, especially in Sea State 5 condition, were measured. The measured relative heave agrees well with theory for Sea States 3 and 4, but is higher for Sea State 5. From Figure 19, we see that the generated wave spectra for the irregular sea has frequencies ranging from ω = 0.4 to 1.8 rad./sec.

---

* R.M.S. = Root Mean Square

However, Figures 20 and 21 show that the measured spectra of the relative surge and sway are concentrated at frequencies lower than 0.4, except in the case of relative surge at Sea State 5, for which a part of the spectrum curve lies between $\omega = 0.4$ and 0.6. Thus, the large R.M.S. values for relative surge and sway result directly from low frequency, or long period motions which are not within the frequency range of the generated wave spectra. Since there is no energy in the wave for those low frequency motions, these motions arise from sources other than those considered in the framework of the mathematical model presented. Therefore, in the comparison between the present theory and experiment, the measured R.M.S. value should be evaluated by eliminating the contribution from the low frequency ($\omega < 0.4$) part. The result obtained by this procedure are also shown in Figure 23 by solid points and it is seen that this leads to better agreement with theory. Although the spectra for relative roll were not measured, it is expected that they would have the same quality as those for sway since these two motions are usually coupled.

The low frequency motions observed in the model tests are probably due to the following reasons:

(1) The ship-mooring line system for the COMET can be thought of as a mass-spring oscillating system which has a dominant resonant frequency in surge. It is shown in Appendix C that the damping ratio of the COMET when performing surge oscillation is practically zero ($C/C_c = 0.0022$) and the natural frequency is of the order of 0.0214 rad./sec. It is therefore

apparent that very little wave energy is needed to excite rather large motions in the vicinity of this frequency. Thus the motion in question could very likely be due to the existence of very slowly decaying transient motions, excited by the initial wave impulses, or of slow variation in drift currents which would be too small to measure when applying the usual procedure for determining the spectral distribution of the wave energy. Such currents could also exist in the real sea although of different degree.

(2) If the center of gravity of the COMET is not exactly in the same vertical plane as the resultant force from the mooring lines (which may be the case in the prototype as well) or if the ship has poor directional stability when oscillating with these low speeds. Then, due to the surging motion, the mooring force may apply yawing moments on the ship and cause the observed relative motion of sway at the sterns and roll.

VI. CONCLUSIONS

On the basis of the extensive theoretical and experimental investigation presented herein, the following important conclusions can be reached:

a. Analytical Results in a Unidirectional Seaway

1. The character of the individual motions of the COMET is roughly the same as those for the PAGE. That is,

i. The surge and pitch motions are relatively independent of heading except in a very small neighborhood of beam seas. This small "window" of headings corresponds to small surge and pitch motions.

ii. The sway and roll motions are both zero in head or following seas. These motions increase monotonically and achieve a maximum value in beam seas.

iii. Heave motions increase monotonically from the value in head seas to about two to three times this value in beam seas.

iv. Yaw motions are zero in head or beam seas. These motions reach a maximum in quartering seas and in a small neighborhood of beam seas, this motion again becomes very small.

2. As a result, orientation in head or following seas minimizes heave, sway, yaw or roll motions; heading in beam seas minimizes surge, yaw and pitch motions.

3. Like the individual ship responses, the relative motions show distinct, narrow "windows" in the local surge, sway and heave responses at the mating juncture in beam seas. The relative roll response is, however, a maximum in the neighborhood of beam seas.

b. Analytical Results in a Non-Unidirectional Seaway

The above results discussed for the case of a unidirectional seaway were used to compute the effect of non-unidirectional seaway with a cosine-squared energy distribution. The computations showed that the narrow "windows" in the surge, yaw and pitch responses disappeared in the case of a non-unidirectional seaway. Slight "dips" in the surge and pitch motion responses do occur in beam seas but these decreases are inconsequential.

As a result, head or following seas are best for relative surge, heave and roll in non-unidirectional seas. For sea states of 4 and below it does not appear to make much difference which heading is chosen with regards to the relative heave motions. However, at Sea State 5, orientation in beam seas becomes the most advantageous.

c. Experimental Results in Regular Waves

In general, the agreement between the model test results and the computed values is quite good. Poor agreement was obtained only in a very few cases and most notably in the sway of the PAGE in beam seas. It is felt, however, that the mathematical model reliably predicts most of the motions of these ships.

d. Experimental Results in Irregular Waves

On the basis of the analytical results, experimental tests were made in irregular head and following seas. According to the theory (as well as from symmetry considerations), there should only be a relative heave and relative surge in this condition. The test results showed, however, that relative sway and roll also existed. The comparison with the theoretical results for the heave relative motions was favorable but that of surge was not. It was apparent from the motion spectra that the discrepancy in relative surge and the existence of relative sway and roll motions was due to factors that were not taken into account by the present theory. These, as discussed in part (b) of

of Section IV, may possibly be: (i) transient or slow variation of the drift of the surface current, and (ii) the eccentricity of the center of gravity of the COMET with the mooring line and/or poor lateral stability at small oscillating speeds.

e. The Effect of Shallow Water on the Relative Motions

A study of the shallow water effects showed that the relative heave motion between the two vessels became smaller and the relative sway and roll became larger as the depth of the water became smaller. The relative surge motion was not sensitive to the variation of water depth. As a result, the criterion for the advantageous location for mating are: (i) If the vertical excursion between two ships is critical, shallow water mating is better; (ii) If the lateral relative motions are critical, deep water mating is advantageous.

VII. RECOMMENDATIONS

It is recommended that:

1. The best heading for mating in a typical short-crested seaway is either head seas or following seas, whichever is more convenient.

2. The existence of the low frequency motion observed in the irregular wave model test may be an overriding consideration in the mating operation. It is recommended that a detailed investigation of this point be made to determine if such undesirable motions can be avoided in practice. Since the frequencies of these motions are extremely low, it is quite likely that

satisfactory manual control of the operation of the PAGE with Voith-Schneider propellers may be possible.

3. It is apparent that any present method for improving the existing mating technique involving the COMET and the PAGE is limited by the inherent characteristics of the two vessels. For example, the roll characteristics of the COMET are essentially those of a conventional dry cargo vessel while the corresponding characteristics of the PAGE are similar to those of a barge, with high initial stability and short rolling periods. Mating problems would be simplified if motion characteristics were essentially the same. To obtain such conditions requires the use of similar hulls, of about the same size, or some radical method of changing motion characteristics. Roll stabilization methods, for example, can change roll amplitudes but will not significantly affect a change in differing roll periods. Short of some radical means of altering basic ship characteristics, e.g., by providing very large ballast capacities, etc., there appears to be no promising method for greatly improving the existing mating method.

Accordingly, for future designs serious consideration should be given to devising other mating systems. Of particular interest is the possibility of docking the PAGE, or a similar beach discharge lighter, in the wet well of a parent ship. It is known, for example, that landing craft can be docked into an LSD type of vessel at considerably higher sea states than would be possible with a conventional mating technique. Considerable experimental and developmental work is currently

underway with regard to such operations, particularly in the case of the FDL and LHA projects, and the results of such studies may be applied to operations involving landing craft as large as the PAGE.

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APPENDIX A

RESULTS OF MODEL TEST IN REGULAR WAVES FROM NETHERLANDS SHIP BASIN

PERIODS

Model 3246^G - COMET						Model 3247^G - PAGE					
Without springs			With springs			Without springs			With springs		
Test no.		Period in sec.	Test no.		Period in sec.	Test no.		Period in sec.	Test no.		Period in sec.
1272	T_φ	16.9	1234	T_φ	16.4	1275	T_φ	5.4	1229	T_φ	5.4
1274	T_ψ	7.7	1236	T_ψ	7.6	1277	T_ψ	6.6	1231	T_ψ	6.5
1273	T_z	7.0	1235	T_z	7.0	1276	T_z	5.2	1230	T_z	5.3
-	T_y	-	1238	T_y	103.2	-	T_y	-	1232	T_y	36.8
-	T_x	-	1237	T_x	57.5	-	T_x	-	1233	T_x	25.0

Wave amplitude = $\bar{h}$ in m (positive: upwards)
Heave amplitude = $\bar{z}$ in m (positive: upwards)
Pitch amplitude = $\bar{\psi}$ in degrees (positive: bow down)
Roll amplitude = $\bar{\varphi}$ in degrees (positive: starboard down)
Surge amplitude = $\bar{x}$ in m (positive: ship forward)
Sway amplitude = $\bar{y}$ in m (positive: ship to port)
Yaw amplitude = $\hat{x}$ in degrees (positive: foreship to port)
ϵ = Phase lag in degrees between the motion of the ship and the wave amplitude (motion prior to wave).

Model 3246^{G} - COMET

Water depth 100 ft. $\alpha = 180^{\circ}$

Test no.	Wave length in m	Wave height in m	Heave in m	Pitch in degr.	Roll in degr.	Surge in m	Sway in m	Yaw in degr.
1239	16	0.535	0	0	0	0	0	0
1241	37	1.23	0	0	0	0	0	0
1242	58	1.94	0.18	0.45	0	0	0	0.06
1243	80	2.67	0.64	0.99	0.08	0.10	0.05	0.19
1244	91.5	3.05	0.81	1.10	1.17*	0.12	0.09	0.26
1245	122	4.07	1.08	5.13	0.37	1.02	0.20	0.33
1246	145	4.07	1.40	5.58	0.69	1.79	0.24	0.49
1247	167.5	4.07	1.51	4.60	0.77	1.91	0.33	0.49
1248	190	4.07	1.80	4.40	0.77	2.32	0.41	0.41
1249	213	4.07	2.33	5.17	1.87	3.22	0.37	0.65

Model 3247^{G} - PAGE

$\alpha = 0^{\circ}$

1239	16	0.535	0	0	0	0	0	0
1241	37	1.23	0.06	0	0.14	0	0	0
1242	58	1.94	0.13	0.19	0.33	0	0	0
1243	80	2.67	0.08	1.28	0.35	0	0	0
1244	91.5	3.05	0.30	2.35	0.27	0.21	0	0.06
1245	122	4.07	1.64	7.41	0.45	1.91	0	0.37
1246	145	4.07	1.99	6.96	0.53	2.60	0.12	0.37
1247	167.5	4.07	2.16	6.19	0.45	2.97	0.24	0.26
1248	190	4.07	2.60	6.06	0.53	3.50	0.28	0.33
1249	213	4.07	3.74	6.02	0.53	4.23	0.16	0.37

*: Period longer than wave period.

Model 3246^G - COMET

Water depth 100 ft. $\alpha = 225$

Test No.	Wave length in m	Wave height in m	Heave in m	Pitch in degr.	Roll in degr.	Surge in m	Sway in m	Yaw in degr.
1250	16	0.535	0	0.10	0.12	0	0	0
1251	37	1.23	0.04	0.10	0.30	0	0	0.07
1252	58	1.94	0.35	0.81	0.54	0	0.08	0.39
1253	80	2.67	0.84	3.04	0.83	0.24	0.13	1.16
1254	91.5	3.05	1.13	4.09	1.40	0.43	0.31	1.68
1255	122	4.07	2.21	6.06	5.41	1.42	1.06	3.34
1256	145	4.07	2.91	5.82	7.73	2.08	1.85	4.11
1257	167.5	4.07	3.50	4.97	9.65	2.24	2.28	4.80
1258	190	4.07	3.54	4.52	14.77	2.73	2.44	5.17
1259	213	4.07	3.28	4.27	24.75	3.34	2.65	5.17

Model 3247^G - PAGE

$\alpha = 45$

1250	16	0.535	0	0	0.07	0	0	0
1251	37	1.23	0.07	0.42	0.43	0	0	0
1252	58	1.94	0.30	2.60	3.30	0.17	0.12	0.50
1253	80	2.67	0.85	5.45	6.19	0.69	0.45	1.36
1254	91.5	3.05	1.27	6.31	4.27	0.88	0.49	1.89
1255	122	4.07	2.77	7.45	6.10	1.83	1.49	3.05
1256	145	4.07	3.19	6.43	5.86	2.44	2.04	3.34
1257	167.5	4.07	3.59	5.33	5.74	2.69	2.40	3.54
1258	190	4.07	4.27	4.76	5.74	2.97	3.17	3.83
1259	213	4.07	4.15	4.56	5.90	3.15	4.03	3.83

Model 3246^{G} - COMET

Water depth 100 ft. α = 270

Test no.	Wave length in m	Wave height in m	Heave in m	Pitch in degr.	Roll in degr.	Surge in m	Sway in m	Yaw in degr.
1260	16	0.535	0	0.02	0	0	0.03	0
1261	37	1.23	0.20	0.39	0.92	0	0.52	0.16
1262	58	1.94	1.13	1.11	1.65	0	1.16	0.49
1263	80	2.67	2.82	1.15	1.68	0.11	1.92	0.80
1264	91.5	3.05	3.66	1.07	1.62	0.21	2.38	0.88
1265	122	4.07	5.19	0.81	0.65	0.33	3.91	0.94
1266	145	4.07	4.80	0.85	1.59	0.45	4.68	0.81
1267	167.5	4.07	4.46	0.94	3.66	0.61	5.62	0.85
1268	190	4.07	4.16	0.94	6.31	0.57	6.06	0.94
1271	213	4.07	4.04	0.98	16.73	0.49	6.06	0.90
1270	241	4.07	3.97	1.02	36.39	0.49	6.02	0.49

Model 3247^{G} - PAGE α = 90

1260	16	0.535	0.04	0	0	0	0	0
1261	37	1.23	0.67	0.22	11.23	0	0.41	0
1262	58	1.94	1.40	0.33	16.45	0	0.60	0
1263	80	2.67	2.21	0.40	13.72	0	1.36	0
1264	91.5	3.05	2.70	0.52	13.69	0	2.01	0
1265	122	4.07	4.12	0.69	15.55	0	4.03	0.11
1266	145	4.07	4.19	0.73	12.54	0	4.72	2.09*
1267	167.5	4.07	4.14	0.65	12.33	0	5.90	0.88*
1268	190	4.07	4.07	0.45	11.11	0.20	6.51	0.66*
1271	213	4.07	4.07	0.33	9.20	0.49	6.72	0.66
1270	241	4.07	4.11	0.28	7.49	0.61	6.59	0.88

*: period longer than wave period

Model 3246^G - COMET

$\alpha = 180^\circ$

Test no.	Wave length in m	Wave height in m	ϵ_{zh} in degr.	$\epsilon_{\psi h}$ in degr.	$\epsilon_{\phi h}$ in degr.	ϵ_{xh} in degr.	ϵ_{yh} in degr.	ϵ_{xh} in degr.
1239	16	0.535	-	-	-	-	-	-
1241	37	1.23	-	-	-	-	-	-
1242	58	1.94	114	360	-	-	-	-
1243	80	2.67	109	308	-	-	-	-
1244	91.5	3.05	93	267	-	-	-	-
1245	122	4.07	54	240	-	68	-	-
1246	145	4.07	55	266	-	95	-	-
1247	167.5	4.07	16	270	-	103	-	-
1248	190	4.07	1	276	-	79	-	-
1249	213	4.07	-11	256	-	78	-	-

Model 3247^G - PAGE

$\alpha = 0^\circ$

1239	16	0.535	-	-	-	-	-	-
1241	37	1.23	-	-	-	-	-	-
1242	58	1.94	167	187	-	-	-	-
1243	80	2.67	9	61	-	-	-	-
1244	91.5	3.05	63	81	-	-122	-	-
1245	122	4.07	34	102	-	- 65	-	-
1246	145	4.07	46	94	-	- 69	-	-
1247	167.5	4.07	15	70	-	-100	-	-
1248	190	4.07	1	70	-	- 98	-	-
1249	213	4.07	- 5	66	-	- 95	-	-

Model 3246^G - COMET

$\alpha = 225^\circ$

Test no.	Wave length in m	Wave height in m	ϵ_{zh} in degr.	$\epsilon_{\psi h}$ in degr.	$\epsilon_{\varphi h}$ in degr.	ϵ_{xh} in degr.	ϵ_{yh} in degr.	ϵ_{xh} in degr.
1250	16	0.535	-	-	-	-	-	-
1251	37	1.23	-	-	-	-	-	-
1252	58	1.94	77	248	-	-	-	277
1253	80	2.67	39	241	190	46	-	215
1254	91.5	3.05	24	246	189	69	64	197
1255	122	4.07	27	272	201	92	96	188
1256	145	4.07	19	267	201	87	87	184
1257	167.5	4.07	11	259	208	89	93	181
1258	190	4.07	27	263	216	80	98	182
1259	213	4.07	5	276	250	93	95	182

Model 3247^G - PAGE

$\alpha = 45^\circ$

1250	16	0.535	-	-	-	-	-	-
1251	37	1.23	178	231	- 5	-	-	-
1252	58	1.94	59	101	- 90	-73	-	160
1253	80	2.67	23	89	- 81	-88	-105	163
1254	91.5	3.05	35	105	- 65	-71	- 85	175
1255	122	4.07	32	104	- 59	-68	- 63	198
1256	145	4.07	22	91	- 75	-67	- 75	195
1257	167.5	4.07	2	89	- 80	-84	-103	182
1258	190	4.07	10	83	- 63	-82	- 91	184
1259	213	4.07	18	71	- 50	-75	- 75	200

Model 3246^0 - COMET

$\alpha = 270^{\circ}$

Test no.	Wave length in m	Wave height in m	ϵ_{zh} in degr.	$\epsilon_{\psi h}$ in degr.	$\epsilon_{\phi h}$ in degr.	ϵ_{xh} in degr.	ϵ_{yh} in degr.	ϵ_{xh} in degr.
1260	16	0.535	-	-	-	-	-	-
1261	37	1.23	-123	-	34	-	42	34
1262	58	1.94	-113	-	28	-	41	37
1263	80	2.67	- 66	-	52	-	68	80
1264	91.5	3.05	- 45	-	54	-	68	109
1265	122	4.07	- 19	-	187	-	78	138
1266	145	4.07	- 16	-	295	-	85	135
1267	167.5	4.07	0	-	296	-	100	138
1268	190	4.07	- 7	-	286	-	87	147
1271	213	4.07	- 15	-	289	-	75	183
1270	241	4.07	- 14	-	352	-	94	-

Model 3247^0 - PAGE

$\alpha = 90^{\circ}$

1260	16	0.535	-	-	-	-	-	-
1261	37	1.23	- 85	-	102	-	225	-
1262	58	1.94	- 28	-	230	-	211	-
1263	80	2.67	- 35	-	230	-	222	-
1264	91.5	3.05	- 20	-	250	-	243	-
1265	122	4.07	- 19	-	262	-	247	-
1266	145	4.07	- 10	-	246	-	246	-
1267	167.5	4.07	0	-	254	-	253	-
1268	190	4.07	- 12	-	257	-	258	-
1271	213	4.07	- 7	-	287	-	259	-
1270	241	4.07	0	-	280	-	265	-

APPENDIX B

TEST RESULTS RECALCULATED FOR UNIT-AMPLITUDE WAVE (1 FOOT)

Model 3246G - COMET

$\beta = 180^{\circ}$

Test no.	Wave length in ft.	Heave in ft.	Pitch in rad.	Roll in rad.	Surge in ft.	Sway in ft.	Yaw in rad.
1239	52.5	0	0	0	0	0	0
1241	121	0	0	0	0	0	0
1242	190	0.0926	0.00123	0	0	0	0.000164
1243	262	0.24	0.00197	0.00016	0.024	0.012	0.00038
1244	300	0.266	0.00191	0.00020	0.0394	0.0295	0.00045
1245	400	0.266	0.00668	0.00048	0.251	0.0492	0.00043
1246	475	0.344	0.00725	0.00030	0.440	0.059	0.00064
1247	550	0.372	0.00597	0.00100	0.470	0.0812	0.00064
1248	624	0.443	0.00572	0.00100	0.570	0.1020	0.00053
1249	700	0.574	0.0067	0.00243	0.792	0.0910	0.00085

Model 3247G - PAGE

$\beta = 0^{\circ}$

1239	52.5	0	0	0	0	0	0
1241	121	0.049	0	0.0000[illegible]	0	0	0
1242	190	0.067	0.00042	0.00091	0	0	0
1243	262	0.03	0.00252	0.00070	0	0	0
1244	300	0.398	0.00410	0.00047	0.07	0	0.0001
1245	400	0.400	0.00960	0.00058	0.47	0	0.00048
1246	475	0.490	0.00975	0.00069	0.64	0.0296	0.00048
1247	550	0.530	0.01364	0.00058	0.73	0.0580	0.00034
1248	624	0.640	0.00785	0.00069	0.86	0.0690	0.00043
1249	700	0.91[illegible]	0.00785	0.00069	1.04	0.0300	0.00048

Model 3246^{0} - COMET

$\beta = 225^{0}$

Test no.	Wave length in ft.	Heave in ft.	Pitch in rad.	Roll in rad.	Surge in ft.	Sway in ft.	Yaw in rad.
1250	52.5	0	0.00099	0.0012	0	0	0
1251	121	0.03[illegible]	0.00043	0.0013	0	0	0.00030
1252	190	0.18	0.00222	0.00148	0	0.04	0.00107
1253	262	0.315	0.00605	0.00165	0 090	0.049	0.00230
1254	300	0.370	0.00710	0.00244	0.141	0.103	0.00292
1255	400	0.543	0.00790	0.00700	0.350	0.260	0.00433
1256	475	0.715	0.00755	0.01000	0.510	0.454	0.00535
1257	550	0.860	0.00645	0.0125	0.550	0.560	0.00625
1258	624	0.870	0.00586	0.0192	0.670	0.600	0.00672
1259	700	0.805	0.00555	0.0322	0.820	0.650	0.00672

Model 3247^{0} - PAGE

$\beta = 45^{0}$

1250	52.5	0	0	0.00070	0	0	0
1251	121	0.057	0.0018	0.00185	0	0	0
1252	190	0.1[illegible]	0 00712	0 00905	0.088	0.06[illegible]	0.0013
1253	262	0 318	0.01070	0.01230	0.[illegible]58	0 [illegible]69	0.0027
1254	300	0.41[illegible]	0 01100	0.0074[illegible]	0.290	0.160	0.0033
1255	400	0 6[illegible]	0.00970	0.00793	0.4[illegible]	0 366	0.0040
1256	475	0.754	0.00840	0.00750	0.600	0.5[illegible]	0.0043
1257	550	0.88[illegible]	0.00695	0.00750	0.66[illegible]	0 590	0.0046
1258	624	1.05[illegible]	0 00620	0.00750	0.730	0.780	0.0050
1259	700	1.[illegible]	0 00590	0.00770	0.774	[illegible]	0.0050

Model 3246^{G} - COMET

$\beta = 270^{\circ}$

Test no.	Wave length in ft.	Heave in ft.	Pitch in rad.	Roll in rad.	Surge in ft.	Sway in ft.	Yaw in rad.
1260	52.5	0	0.00020	0	0	0.056	0
1261	121	0.163	0.00170	0.00400	0	0.423	0.00069
1262	190	0.582	0.00304	0.00452	0	0.597	0.00134
1263	262	1.060	0.00230	0.00330	0.041	0.720	0.00160
1264	300	1.200	0.00186	0.00280	0.069	0.778	0.00153
1265	400	1.270	0.00105	0.00084	0.081	0.960	0.00121
1266	475	1.180	0.00110	0.00207	0.110	1.150	0.00105
1267	550	1.100	0.00121	0.00475	0.150	1.380	0.00110
1268	624	1.020	0.00121	0.00820	0.140	1.490	0.00121
1271	700	0.990	0.00127	0.02170	0.120	1.490	0.00115
1270	790	0.980	0.00130	0.04710	0.120	1.480	0.00064

Model 3247^{G} - PAGE

$\beta = 90^{\circ}$

1260	52.5	0.075	0	0	0	0	0
1261	121	0.540	0.00095	0.0485	0	0.33	0
1262	190	0.721	0.00090	0.0450	0	0.31	0
1263	262	0.830	0.00080	0.0272	0	0.51	0
1264	300	0.885	0.00090	0.0238	0	0.66	0
1265	400	1.020	0.00090	0.0202	0	0.99	0.00015
1266	475	1.040	0.00095	0.0162	0	1.16	0.00272*
1267	550	1.020	0.00085	0.0160	0	1.45	0.00113*
1268	624	1.000	0.00058	0.0144	0.049	1.60	0.00085*
1271	700	1.000	0.00043	0.0120	0.120	1.65	0.00085*
1270	790	1.010	0.00036	0.0097	0.150	1.62	0.00113

*: period longer than wave period.

APPENDIX C

EFFECT OF THE MOORING LINE ON THE SURGE MOTION OF COMET

Here we examine the effect of the stiffness of mooring line on the surge motion of COMET in head sea at Sea State 5. The ship-mooring line system is considered as a single degree mass-spring oscillating system. We calculate for the following aspects:

(a) The damping ratio at resonance. The spring constant k_x for a 55 pounds per foot mooring line at head sea is estimated, by using Figure 24 (taken from Reference (13)) to be 440 pounds per foot. The mass of COMET is 9.65×10^5, so the natural frequency ω_n of this system is $\sqrt{440/9.65 \times 10^5} = 0.0214$ radians per second and the critical damping of the system is $2\sqrt{440 \times 9.65 \times 10^5} = 41300$ pounds per foot per second. The hydrodynamic damping for unit amplitude wave at the natural frequency is estimated by Equation [19] to be 69.5 pounds per foot per second. To estimate the damping due to frictional force in model test, we considered the fact that the measured R.M.S. value of relative surge of 8 feet at Sea State 5 was almost solely due to the motion of the COMET and obtained the frictional damping to be approximately 21 pounds per foot per second. Therefore, the damping ratio of the system is 0.0022 which is, of course, very small.

(b) The amplitude of motion at resonance. To find the response of COMET due to unit amplitude wave at resonant frequency we use the equation of motion for surge alone, or

$$m_{\ddot{x}} + X_{\dot{x}}\dot{x} + k_x x = -\rho\omega^2 e^{-\frac{2\pi\bar{h}}{\lambda}} \left[\int_{\xi_s}^{\xi_b} S\, e^{-i\frac{2\pi\xi}{\lambda}} d\xi \right] e^{i\omega t}$$

which gives the amplitude of surge at ω_n to be 265 feet or a R.M.S. value of 186 feet.

(c) Wave energy needed to excite the motion. Since the relation between the amplitudes of ship motion and wave may be assumed to be linear, the R.M.S. value of the amplitude of the wave which will produce a motion of 8 feet is estimated to be 8/186 = 0.043 ft. The energy such a wave carries is then $2(0.043)^2 = 0.0037\ \text{ft}^2$. If we distribute the energy over a frequency band of 0.02 radians per second width we have a height of the spectral density curve at this frequency of about 0.195 ft^2sec. From Figure 19 the total energy of the measured wave spectrum for sea state 5 is estimated to be 12.7 ft^2 and the maximum height of the spectral density is seen to be $1.2 \times 3.28^2 = 13\ \text{ft}^2$ sec. Thus, we see that the wave energy needed to excite the resonance motion is only 0.029 percent of the total energy measured in the test basin at 1.4 percent for the spectral density.

From the above calculations and the low damping ratio in this system we see that only a small part of the wave energy in the test basin is needed to excite a rather large motion in the vicinity of the mooring-line-induced, surge resonant frequency. Such low energy is quite hard to measure when applying the usual procedure for determining the spectral distribution of the wave

energy. Furthermore, the computation assumed the energy in the sea was represented by Neumann's spectrum as given by Equation [63]. At ω_n and sea state 5 this gives a value of $A^2(\omega_n) = 0.315 \times 10^{-26}$ $ft^2 sec$, which is practically zero and therefore does not show up in the computation. However, this does not mean that the resonance effect can not exist in the real sea since it is almost certain that the Neumann spectrum or any other seaway spectrum would not be sufficiently precise to account for such a small part of the total wave energy as 0.029 percent.

TABLE 1

Characteristic Dimensions of Ships

Title	Unit	COMET	PAGE
Length	ft.	475	300
Breadth	ft.	78	65
Draft	ft.	22	7
Mass	slug	9.65×10^{5}	1.63×10^{5}
Distance from bow to c.g.	ft.	245	157
Distance from c.b. to c.g., $\overline{BG}$	ft.	18.73	12
Distance from water surface to c.g., $\overline{OG}$	ft.	8.8	9.2
Metacentric height	ft.	3.83	46
Total roll moment of inertia I_{xt}	slug ft^2	8×10^{8}	1.53×10^{8}
Moment inertia of pitch I_y	slug ft^2	1.385×10^{10}	9.21×10^{8}
Moment inertia of yaw I_z	slug ft^2	1.385×10^{10}	9.21×10^{8}
Roll period	sec.	16.5	5.4
Pitch period	sec.	7.6	6.5
Heave period	sec.	7.0	5.3

TABLE 2

Three Dimensional Damping Factor

For COMET

Frequency ω	For Heave C_z	For Pitch C_θ	For Sway C_y	For Yaw C_ψ
0.4	0.4	0.04	0.2	0.01
0.5	0.6	0.07	0.3	0.01
0.6	0.8	0.14	0.44	0.05
0.7	0.98	0.30	0.59	0.14
0.8	1.08	0.60	0.69	0.26
0.9	1.12	0.92	0.75	0.40
1.0	1.07	1.12	0.79	0.53
1.1	1.02	1.16	0.83	0.62
1.2	0.98	1.12	0.86	0.69
1.3	0.96	1.06	0.88	0.73
1.4	1.00	1.00	0.89	0.76

For PAGE

Frequency ω	For Heave C_z	For Pitch C_θ	For Sway C_y	For Yaw C_ψ
0.4	0.60	0.07	0.35	0.03
0.5	0.82	0.16	0.47	0.04
0.6	1.04	0.43	0.63	0.09
0.7	1.12	0.88	0.75	0.19
0.8	1.07	1.12	0.81	0.22
0.9	1.02	1.16	0.86	0.47
1.0	0.96	1.09	0.89	0.59
1.1	1.00	1.00	0.92	0.68
1.2	1.00	1.00	0.94	0.74
1.3	1.00	1.00	0.95	0.77
1.4	1.00	1.00	0.96	0.80

TABLE 3

Comparison Between Theory and Experiment
at 100 Feet Water Depth

Ship	Motion	Heading β°	Amplitude	Phase
COMET	Surge	180	Excellent	Good
		225	Good	Good
		270	Fair	No Data
	Heave	180	Good	Good
		225	Fair	Good
		270	Fair	Good
	Pitch	180	Excellent	Good
		225	Good	Good
		270	Fair	No Data
	Sway	180	Fair	No Data
		225	Excellent	Excellent
		270	Fair	Excellent
	Roll	180	Good	No Data
		225	Good	Fair
		270	Fair	Fair
	Yaw	180	Good	No Data
		225	Fair	Good
		270	Good	Good

TABLE 3 (Concluded)

Ship	Motion	Heading β^o	Amplitude	Phase
PAGE	Surge	0	Excellent	Good
		45	Excellent	Excellent
		90	Good	No Data
	Heave	0	Excellent	Good
		45	Good	Good
		90	Good	Good
	Pitch	0	Excellent	Excellent
		45	Fair	Good
		90	Excellent	No Data
	Sway	0	Good	No Data
		45	Fair	Excellent
		90	Poor	Good
	Roll	0	Excellent	No Data
		45	Excellent	Excellent
		90	Excellent	Excellent
	Yaw	0	Good	No Data
		45	Fair	Excellent
		90	Data not Reliable	No Data

Note: Poor - Do not agree in quality and quantity.
Fair - Agree qualitatively, but not close enough in quantity.
Good - Agree qualitatively, not far off in quantity.
Excellent - Agree well in quality and quantity.

TABLE 4

Experimental R.M.S. Values of Relative Motion
Between COMET and PAGE in Irregular Sea

(a) Mooring Line Weight 55 lbs/ft.

Mean of 1/3 Highest Waves, ft.	Motions	R.M.S. Value, σ			
		Measured		After Correction	
		Head Sea	Following Sea	Head Sea	Following Sea
3.88	X_R	1.11	1.38	0	0
	Y_R	0.066	0.368	0	0
	Z_R	0.72	0.82	0.72*	0.82
	φ_R	0.00094	0.0021	—	—
5.02	X_R	5	2.08	0	0
	Y_R	0.93	0.53	0	0
	Z_R	1.4	1.54	1.40	1.54
	φ_R	—	—	—	—
5.65	X_R	2.32	2.22	0	0
	Y_R	0.69	0.805	—	—
	Z_R	1.57	1.75	1.57	1.75
	φ_R	0.0033	0.00227	—	—
10.1	X_R	7.13	8.05	0	0
	Y_R	2.7	4.9	0	0
	Z_R	4.53	5.15	4.53	5.15
	φ_R	0.0094	0.0089	—	—

* No correction is needed for relative heave.

TABLE 4 (Concluded)

(b) Mooring Line Weight 18 lbs/ft.

Mean of 1/3 Highest Waves, ft.	Motions	R.M.S. Value, σ			
		Measured		After Correction	
		Head Sea	Following Sea	Head Sea	Following Sea
3.88	X_R	0.621	0.458	0	0
	Y_R	0.163	0.459	0	0
	Z_R	0.46	0.59	0.46	0.59
	φ_R	0.00094	0.0021	—	—
5.65	X_R	3.5	2.56	0	0
	Y_R	1.21	0.915	0	0
	Z_R	1.44	1.67	1.44	1.67
	φ_R	0.0033	0.00227	—	—
10.1	X_R	10.0	7.03	3.44	0
	Y_R	5.77	3.21	0	0
	Z_R	3.44	4.51	3.44	4.51
	φ_R	0.0094	0.0089	—	—

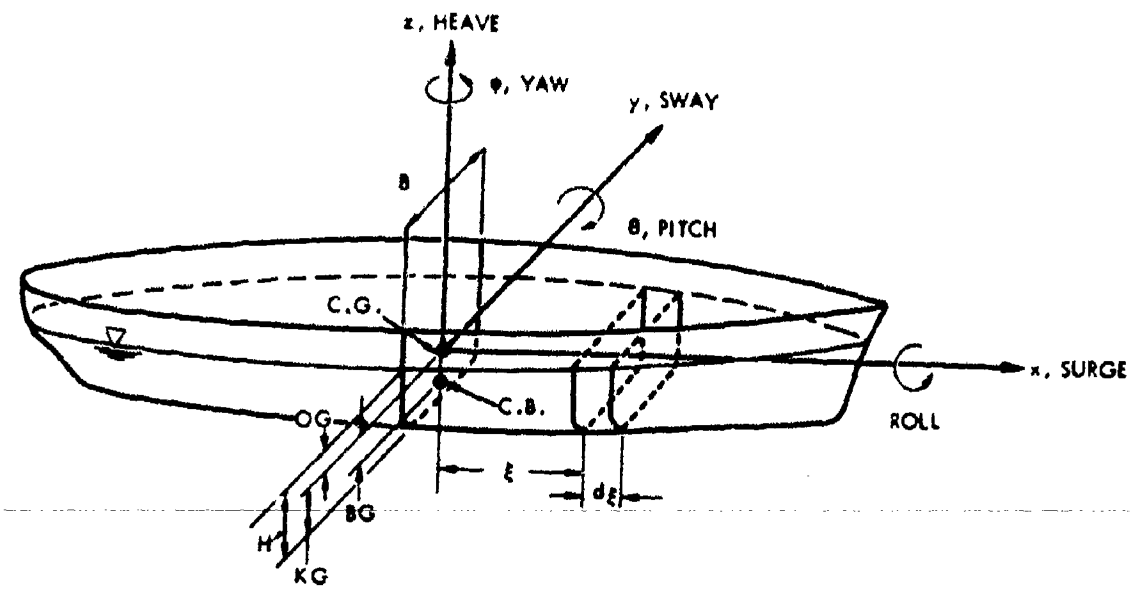


FIGURE 1 - DEFINITION SKETCH OF SHIP MOTIONS

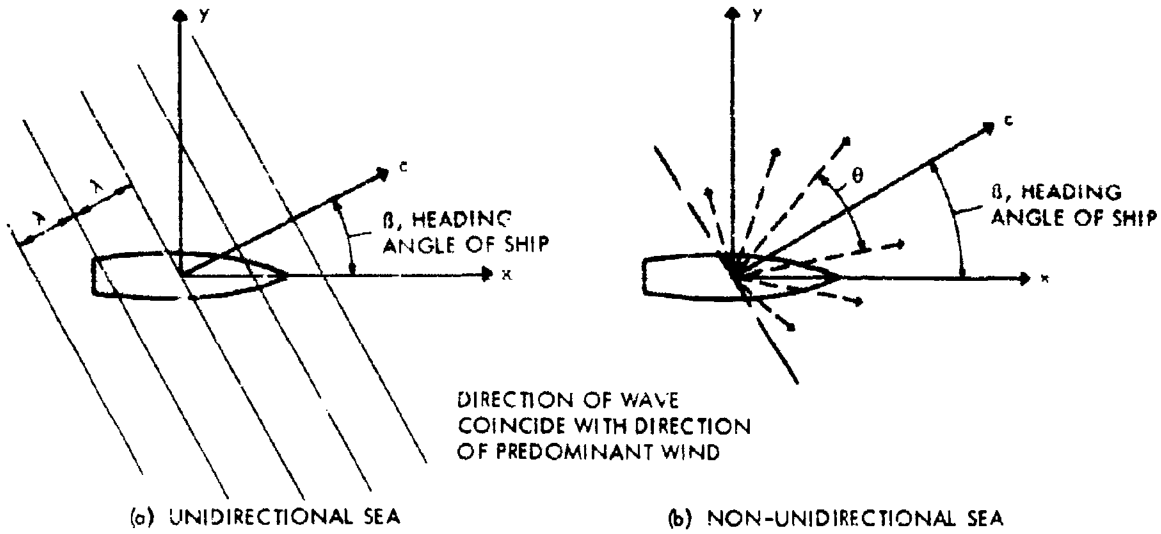
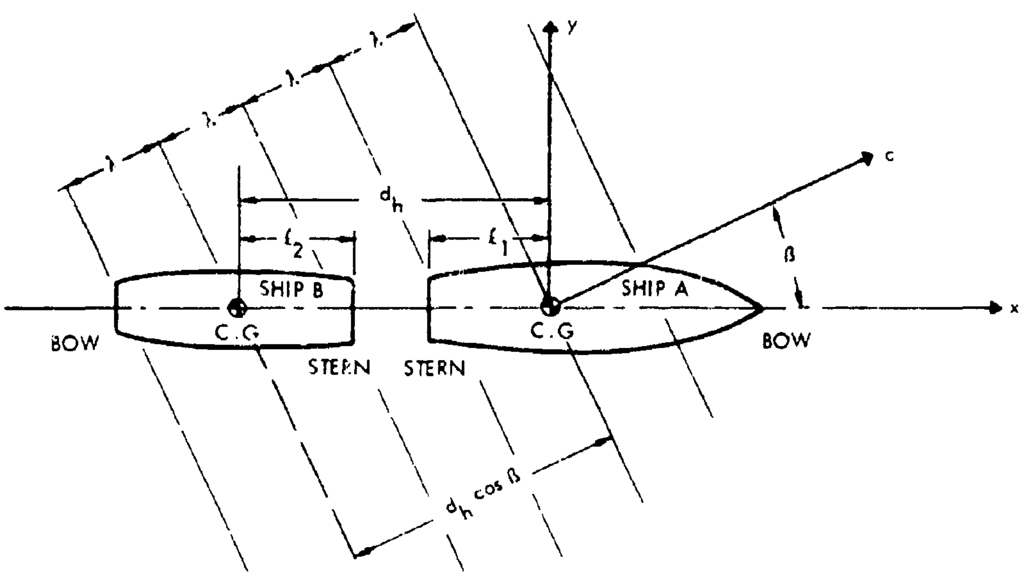
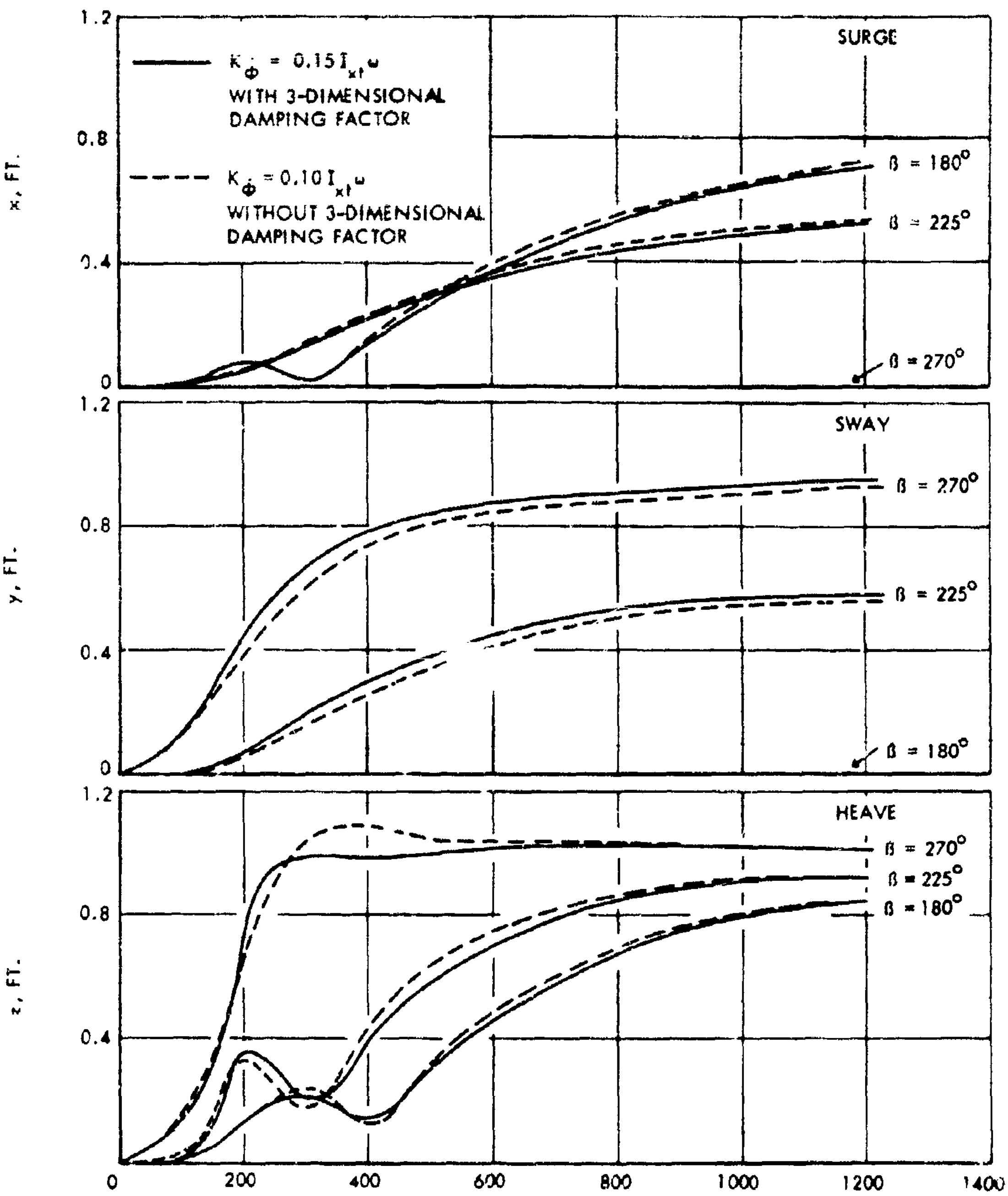


FIGURE 2 - DEFINITION SKETCH OF SHIP HEADING AND WAVE DIRECTION



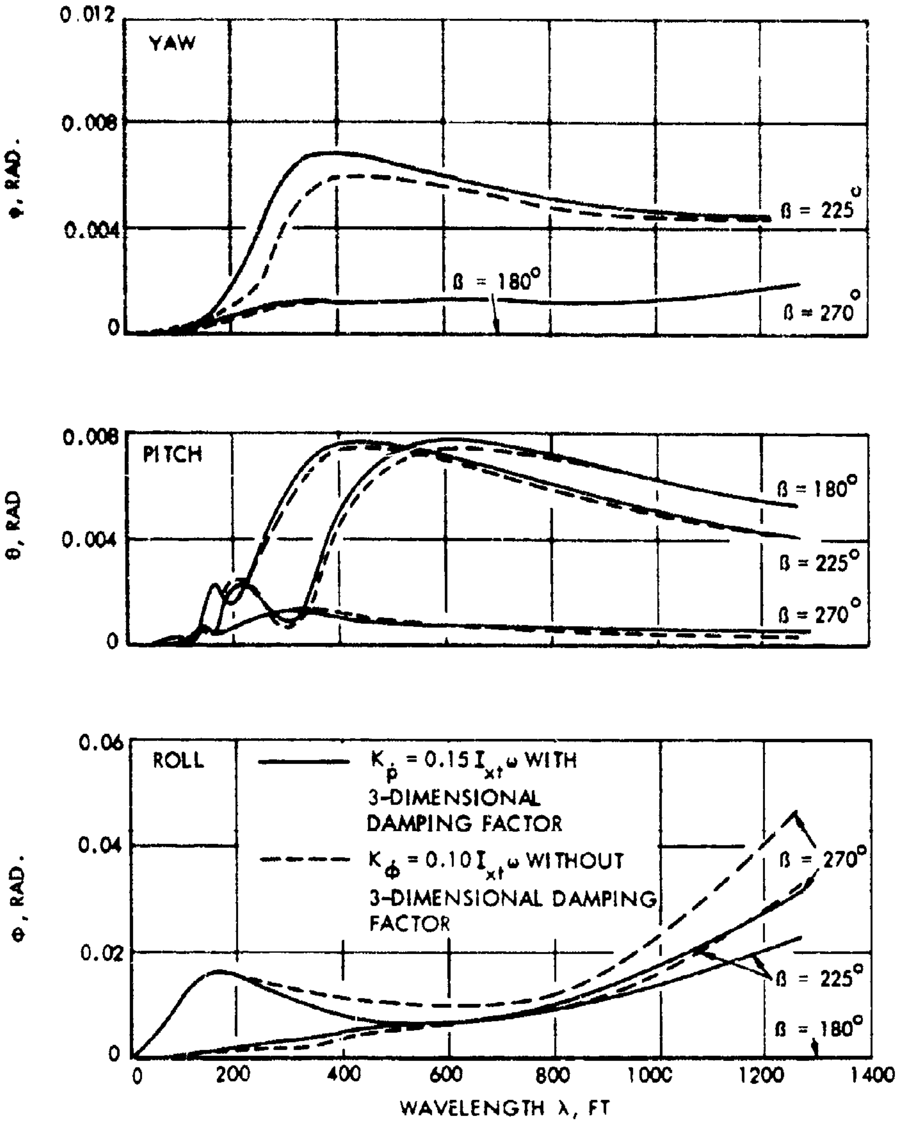
$$\text{PHASE DIFFERENCE } \delta = \frac{d_h \cos \beta}{\lambda} \cdot 2\pi$$

FIGURE 3 - THE PHASE DIFFERENCE BETWEEN TWO SHIPS WITH RESPECT TO THE PHASE OF WAVE IN OBLIQUE SEAS



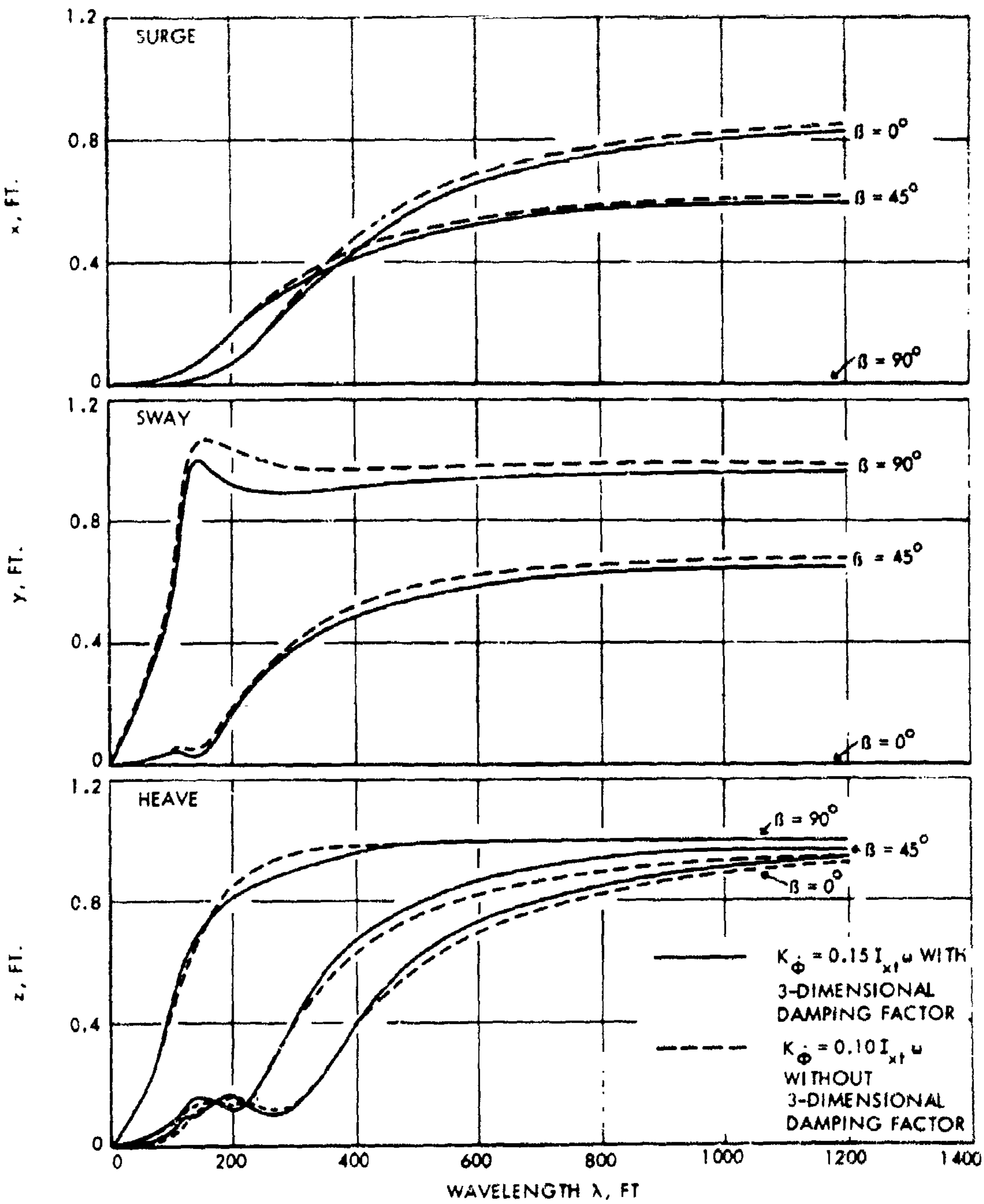
(a) SURGE, SWAY AND HEAVE

FIGURE 4 - RESPONSES OF COMET DUE TO UNIT-AMPLITUDE, REGULAR WAVES



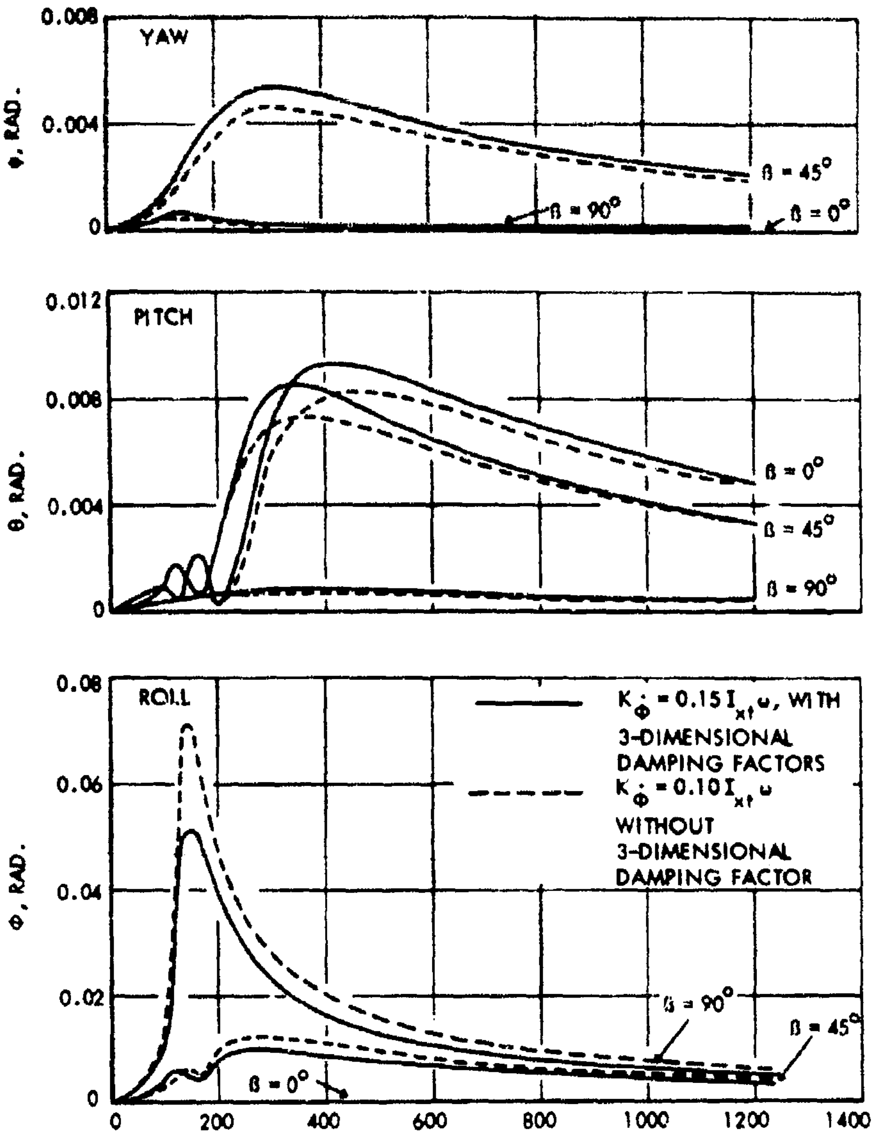
(b) YAW, PITCH AND ROLL

FIGURE 4 - (CONCLUDED)



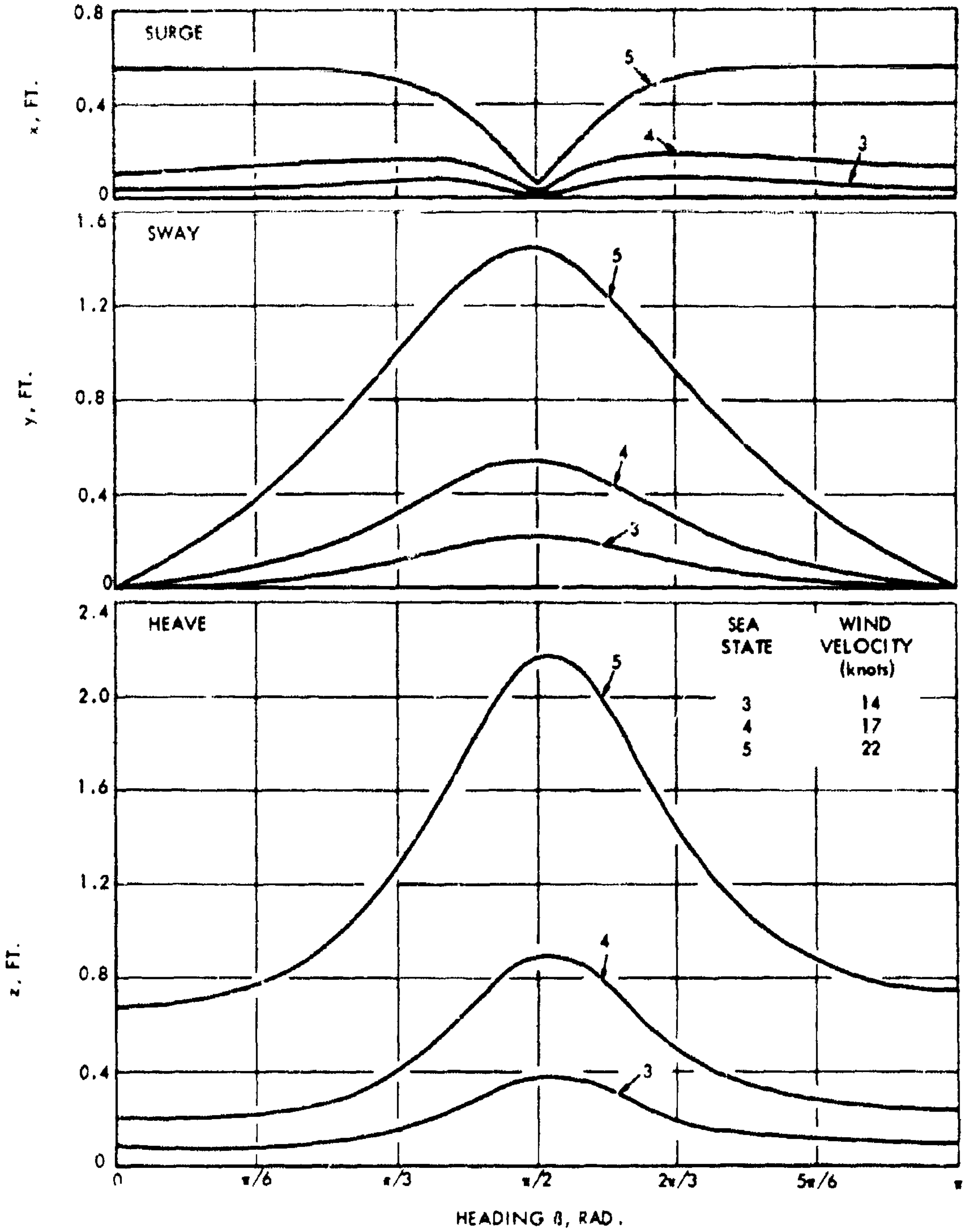
(a) SURGE, SWAY AND HEAVE

FIGURE 5 - RESPONSES OF PAGE DUE TO UNIT-AMPLITUDE, REGULAR WAVES



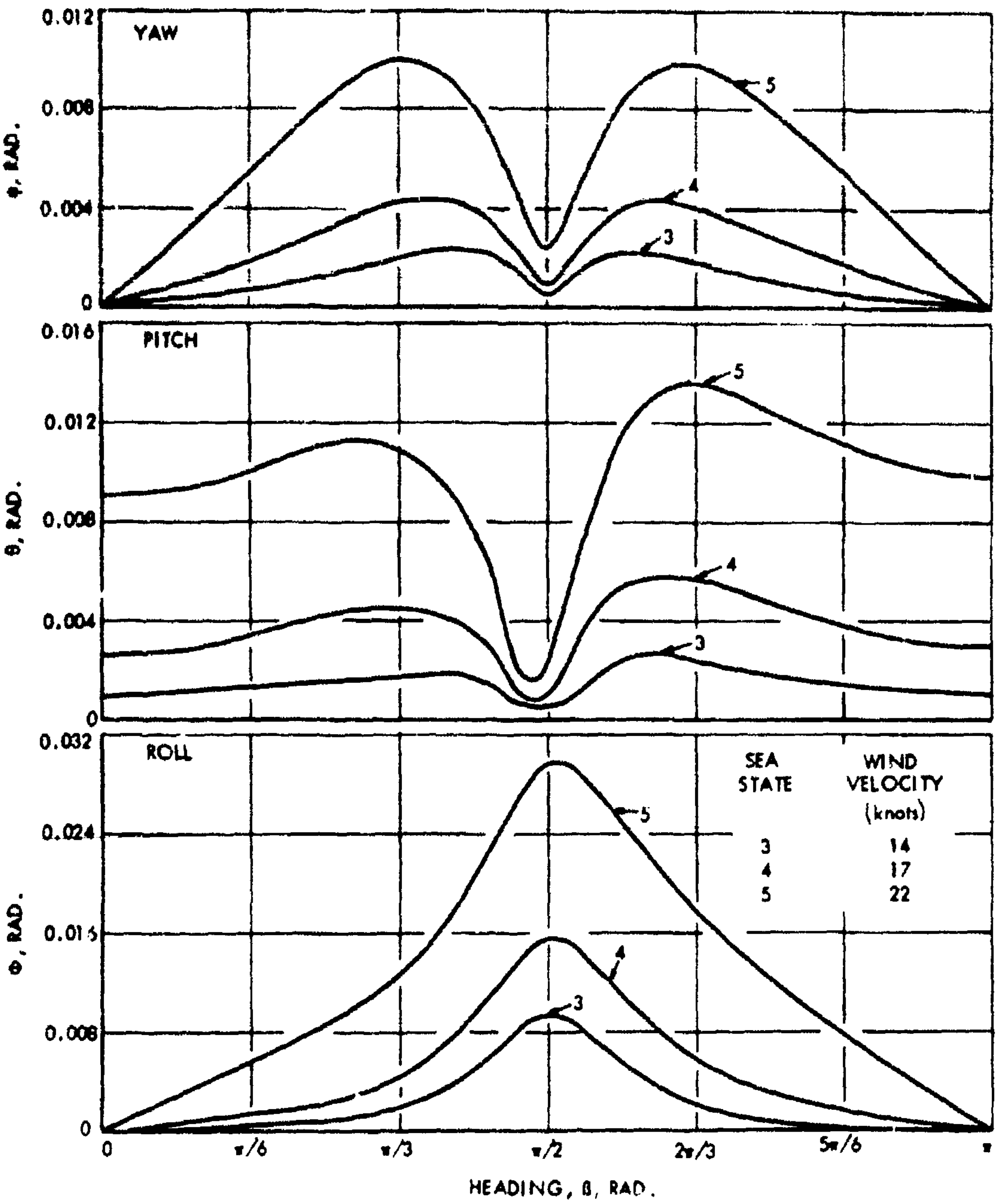
(b) YAW, PITCH AND ROLL

FIGURE 5 - (CONCLUDED)



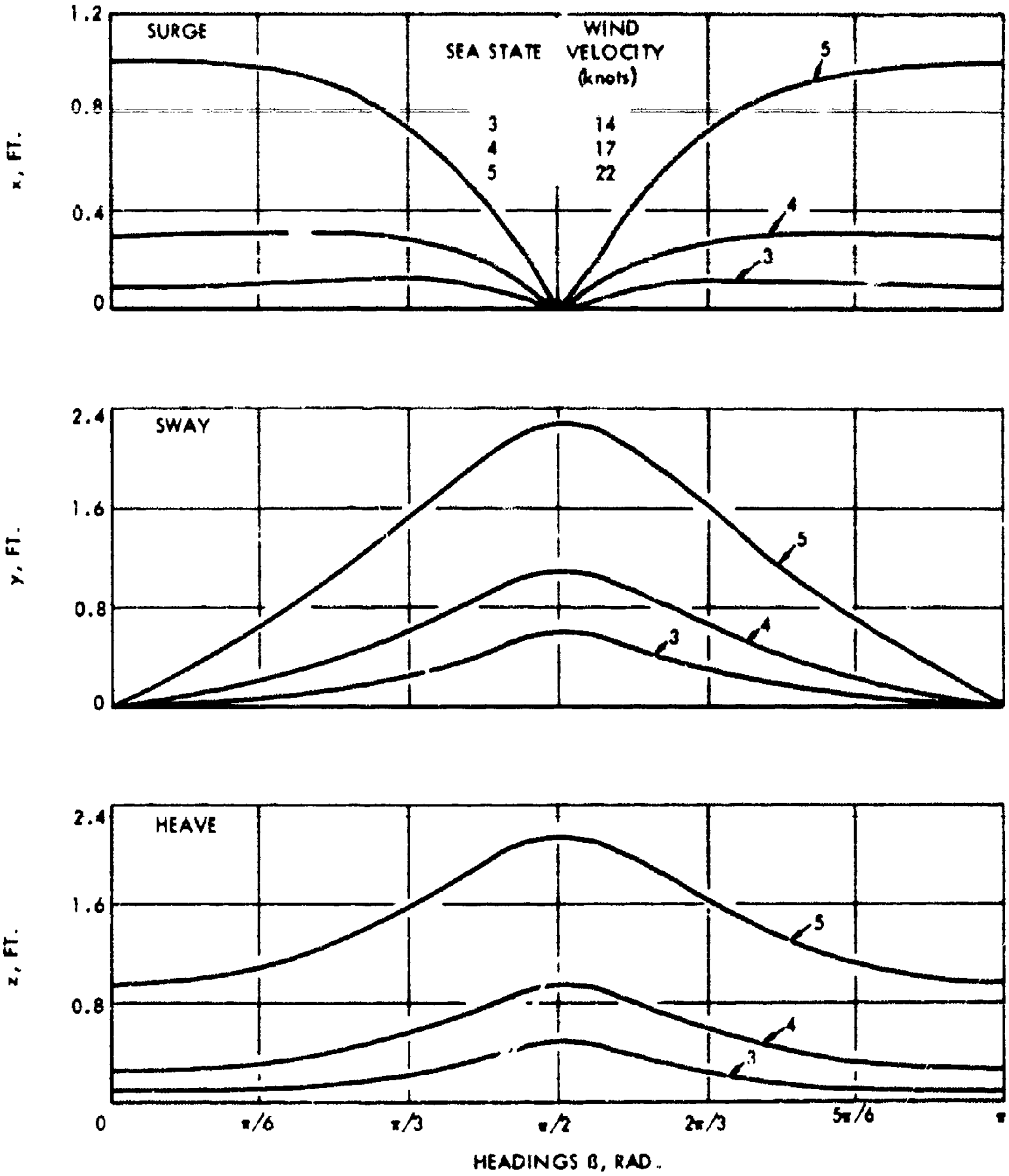
(a) SURGE, SWAY AND HEAVE

FIGURE 6 - R.M.S. VALUES OF MOTIONS FOR COMET IN UNIDIRECTIONAL SEA



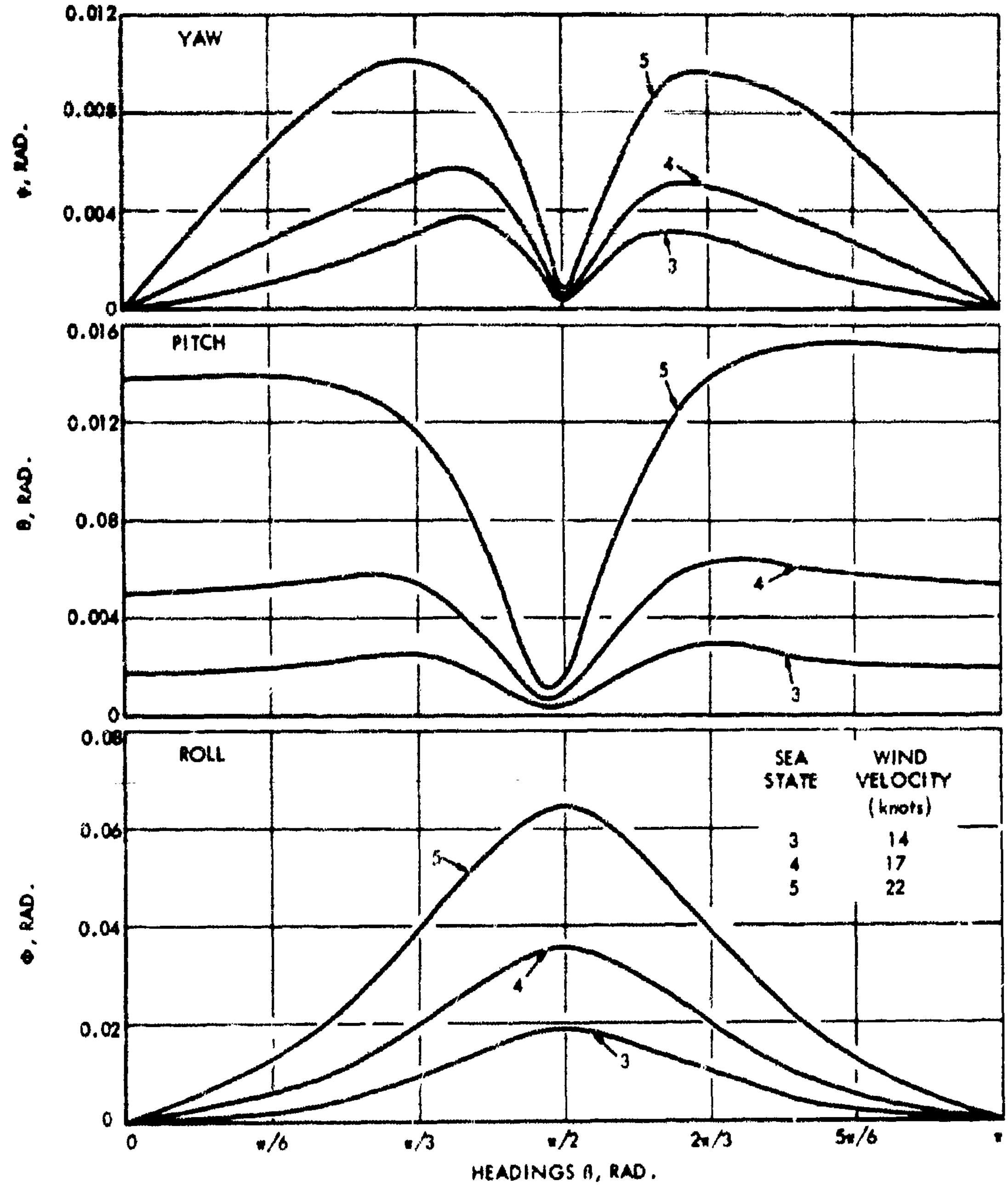
(b) YAW, PITCH AND ROLL

FIGURE 6 - (CONCLUDED)



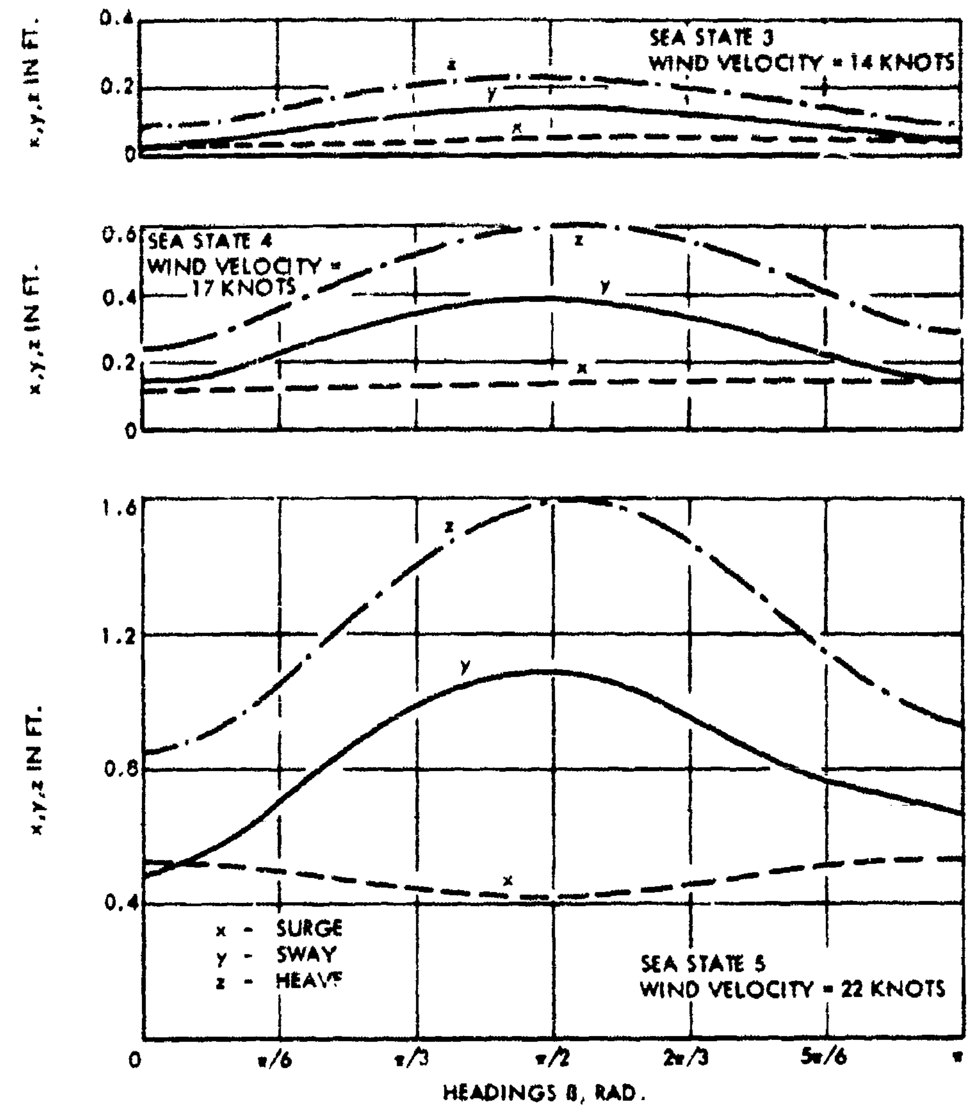
(a) SURGE, SWAY AND HEAVE

FIGURE 7 - R.M.S. VALUES OF MOTIONS FOR PAGE IN UNIDIRECTIONAL SEA



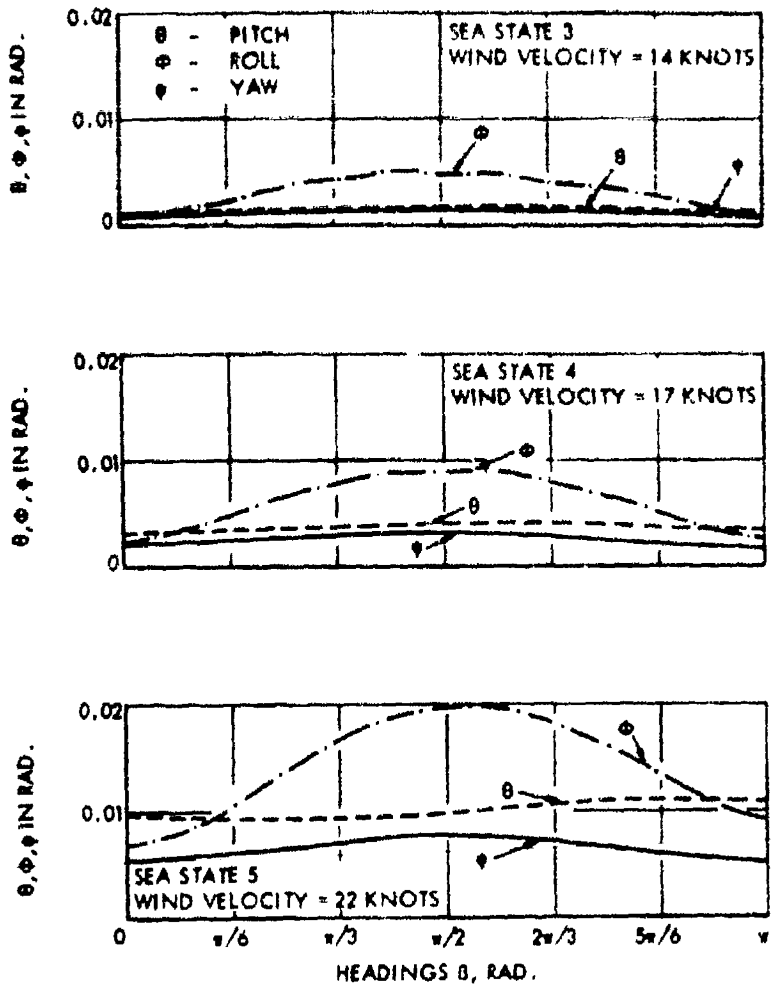
(b) YAW, PITCH AND ROLL

FIGURE 7 - (CONCLUDED)



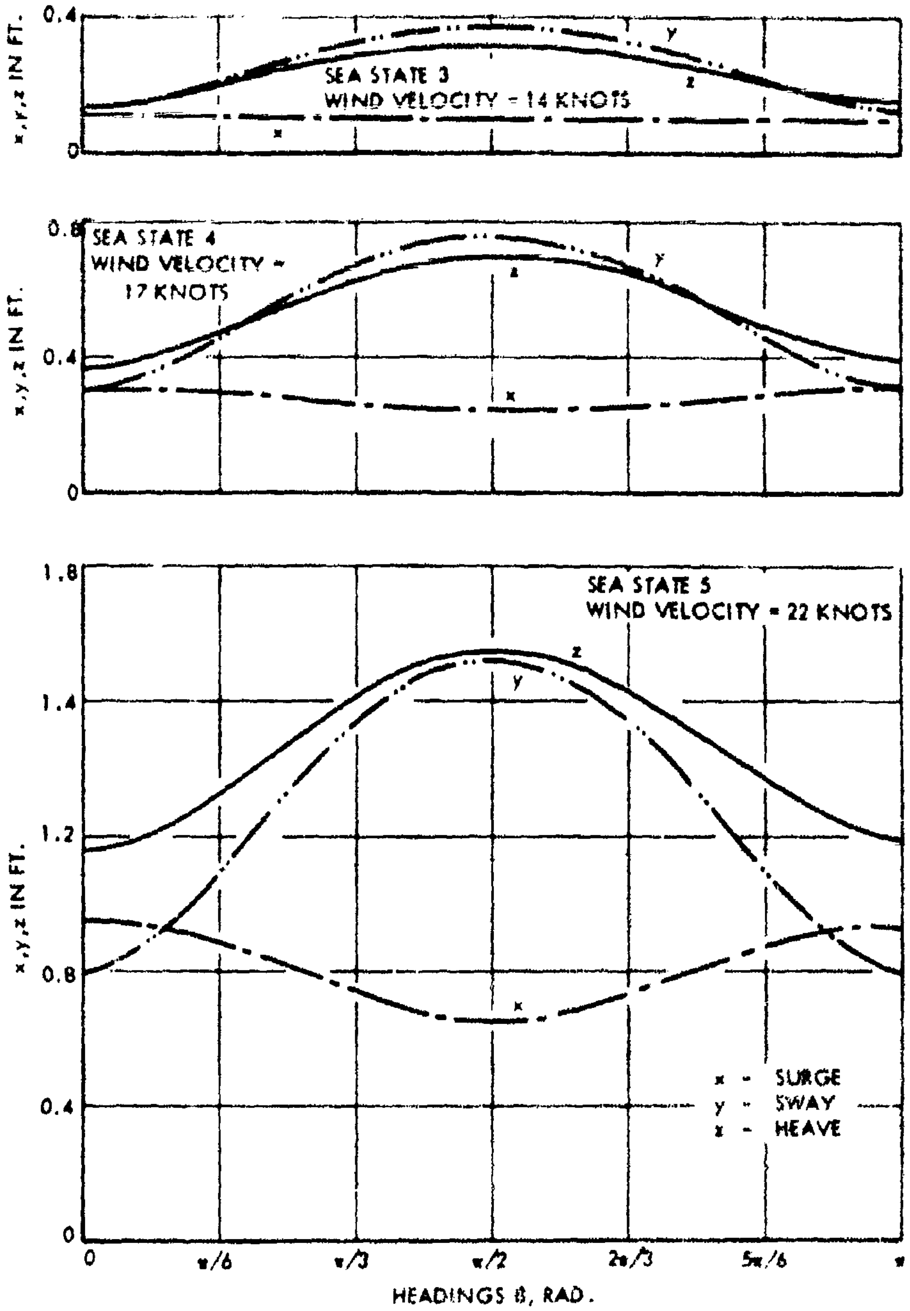
(a) SURGE, SWAY AND HEAVE

FIGURE 8 - R.M.S. VALUES OF MOTIONS FOR COMET IN NON-UNIDIRECTIONAL SEA



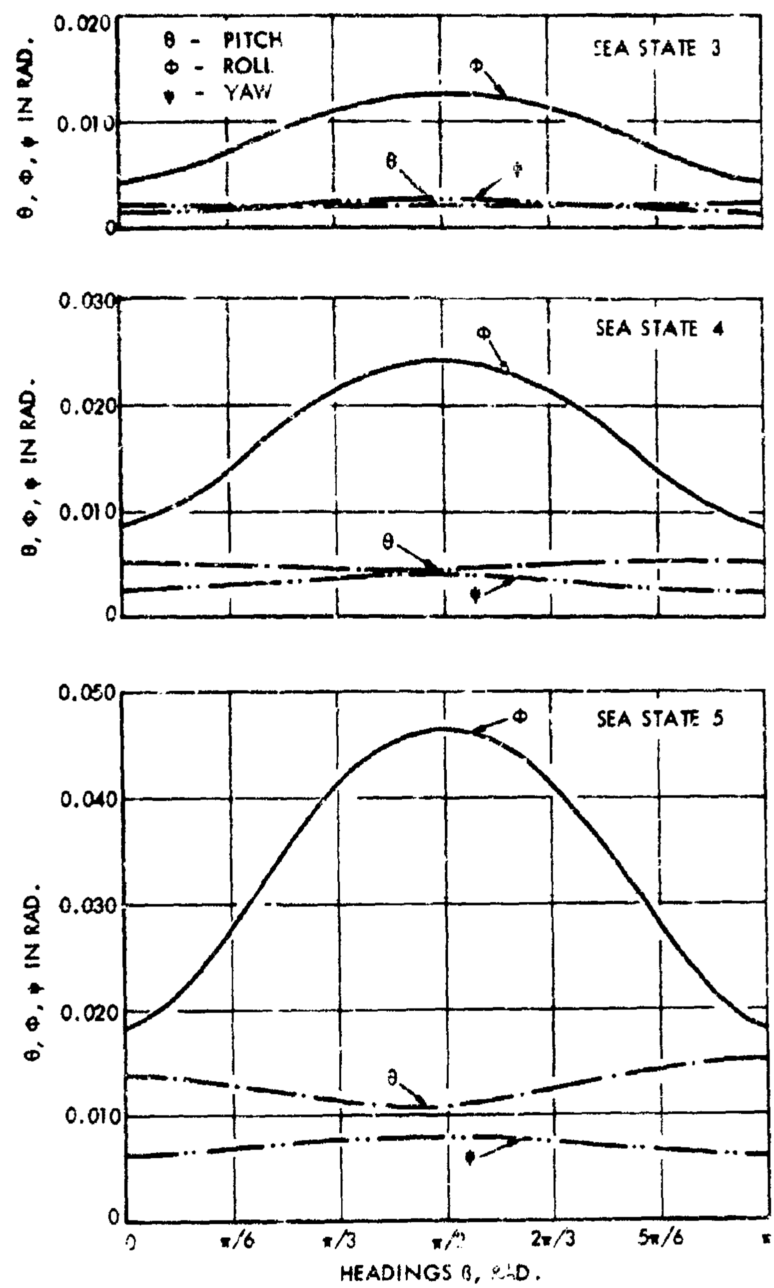
(b) YAW, PITCH AND ROLL

FIGURE 8 - (CONCLUDED)



(a) SURGE, SWAY AND HEAVE

FIGURE 9 - R.M.S. VALUES OF MOTIONS FOR PAGE IN NON-UNIDIRECTIONAL SEA



(b) PITCH, ROLL AND YAW

FIGURE 9 - (CONCLUDED)

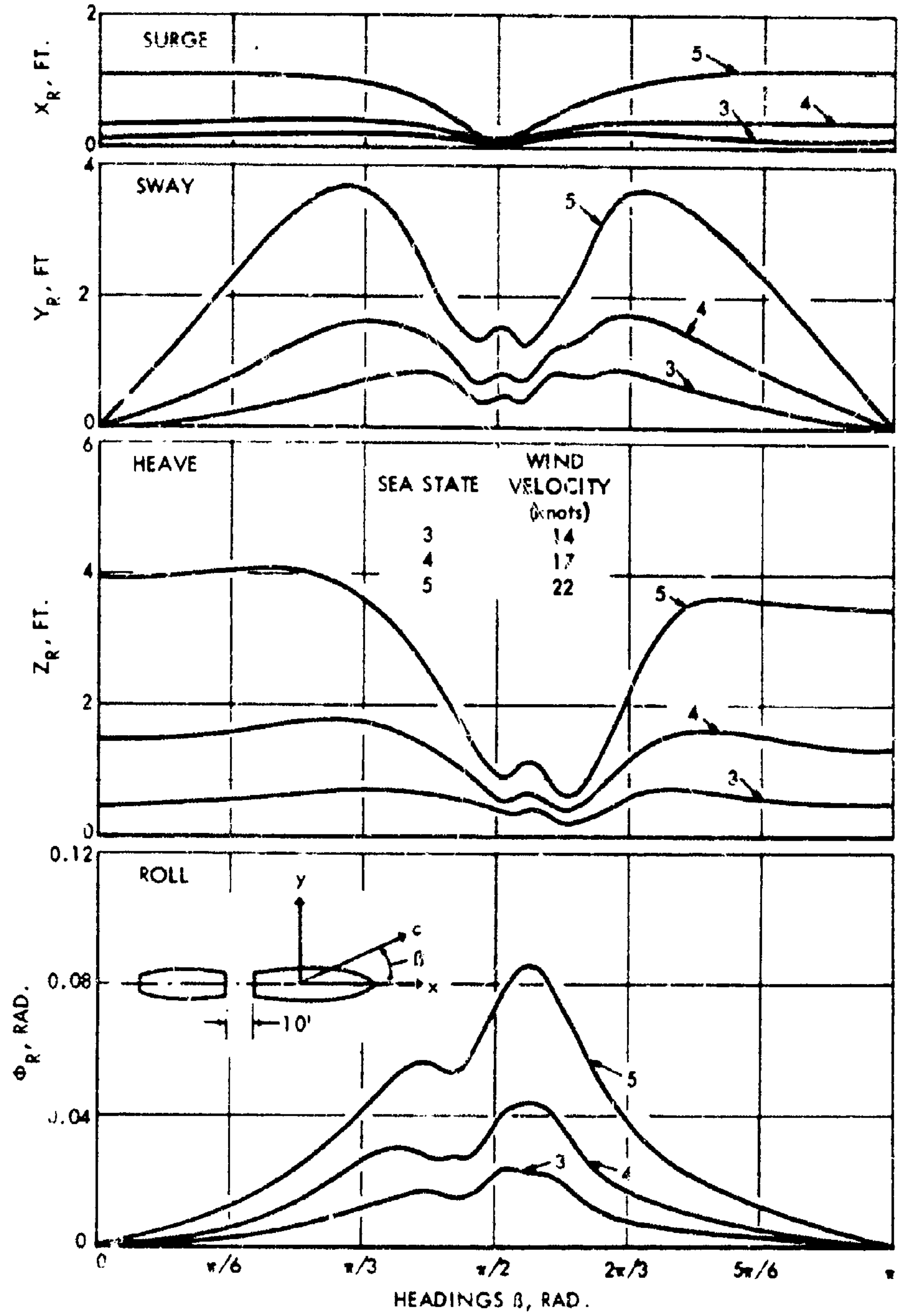


FIGURE 10 - R.M.S. VALUES OF RELATIVE MOTIONS BETWEEN COMET AND PAGE IN UNIDIRECTIONAL SEA

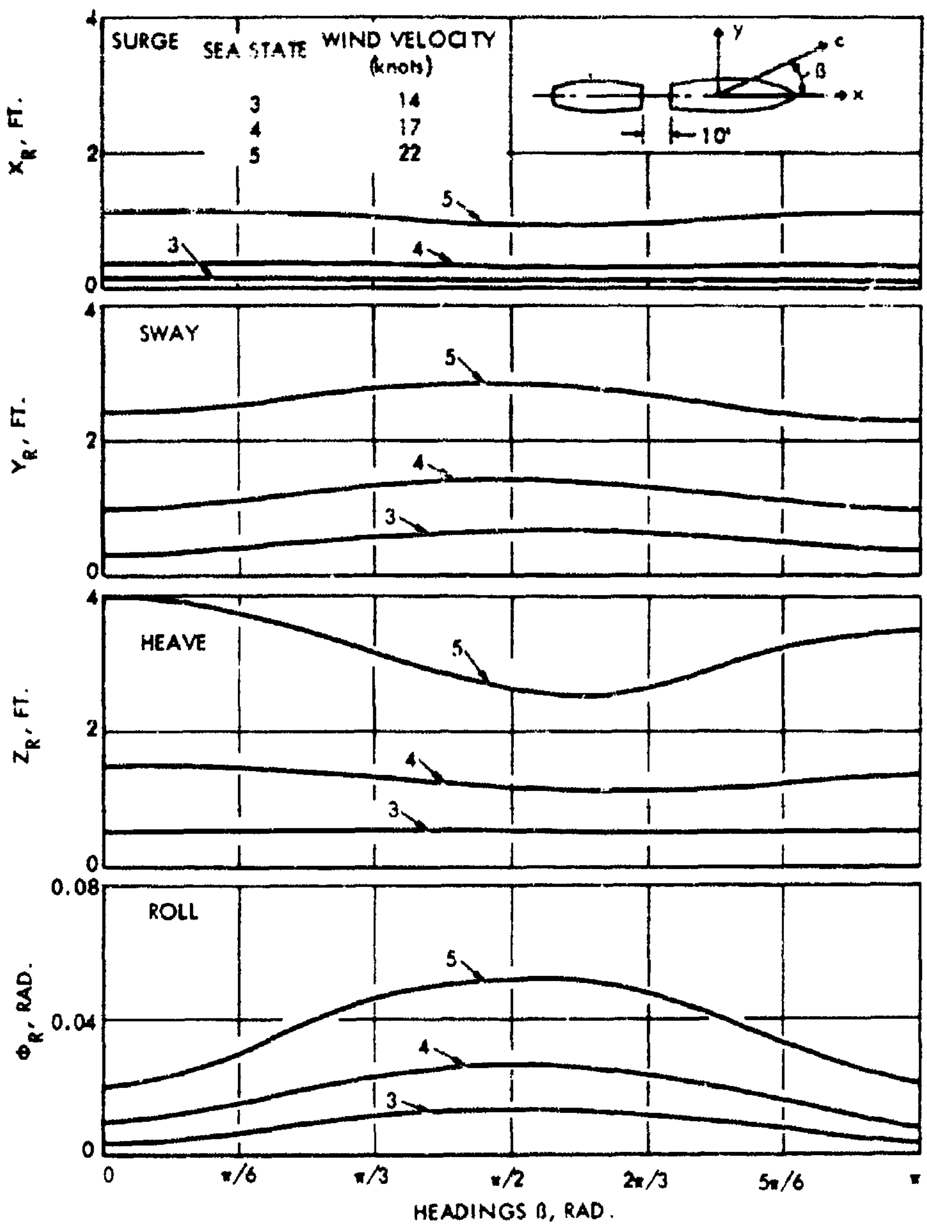


FIGURE 11 - R.M.S. VALUES OF RELATIVE MOTIONS BETWEEN COMET AND PAGE IN NON-UNIDIRECTIONAL SEA

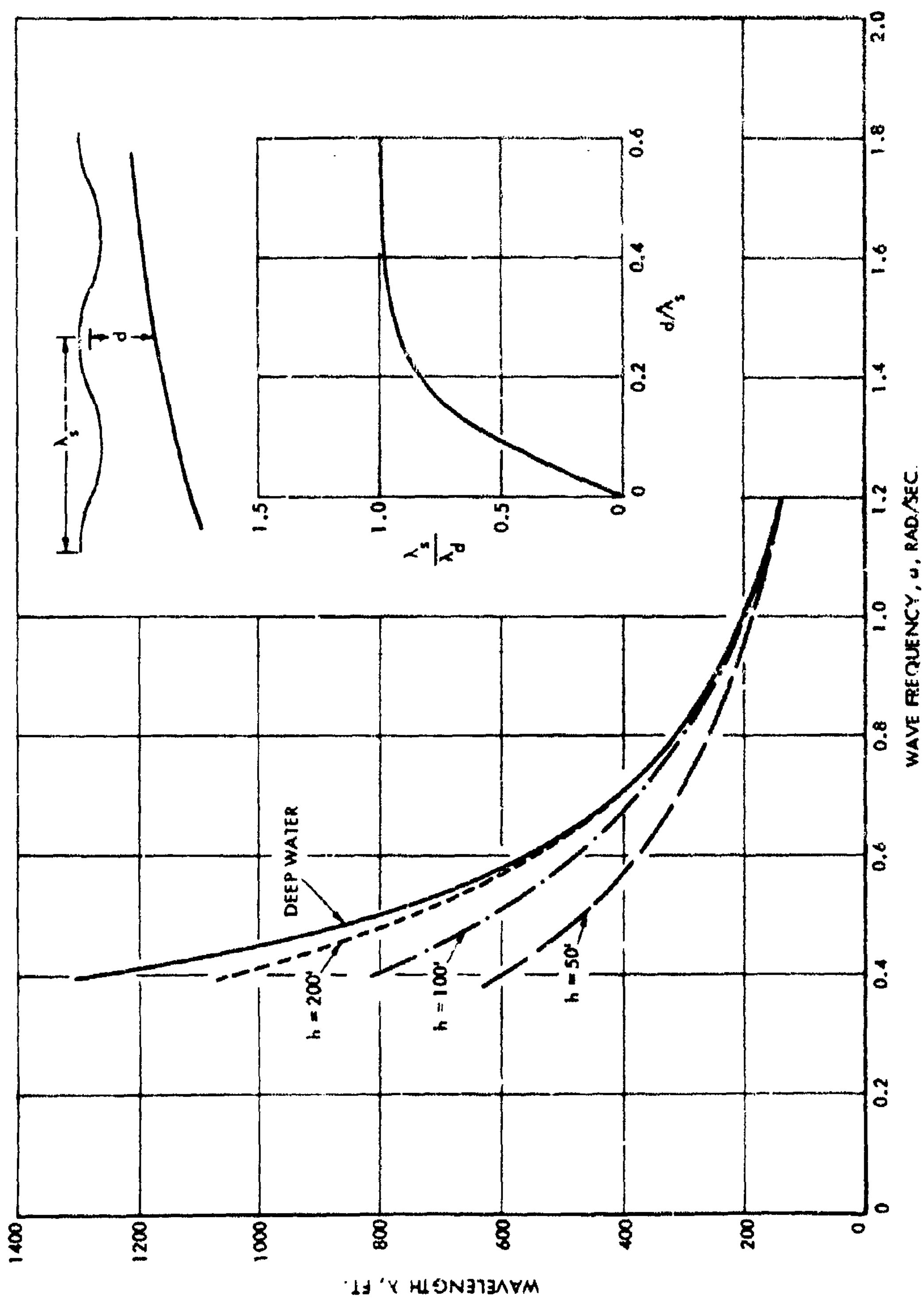


FIGURE 12 - VARIATION OF WAVELENGTH DUE TO SHALLOW WATER EFFECT

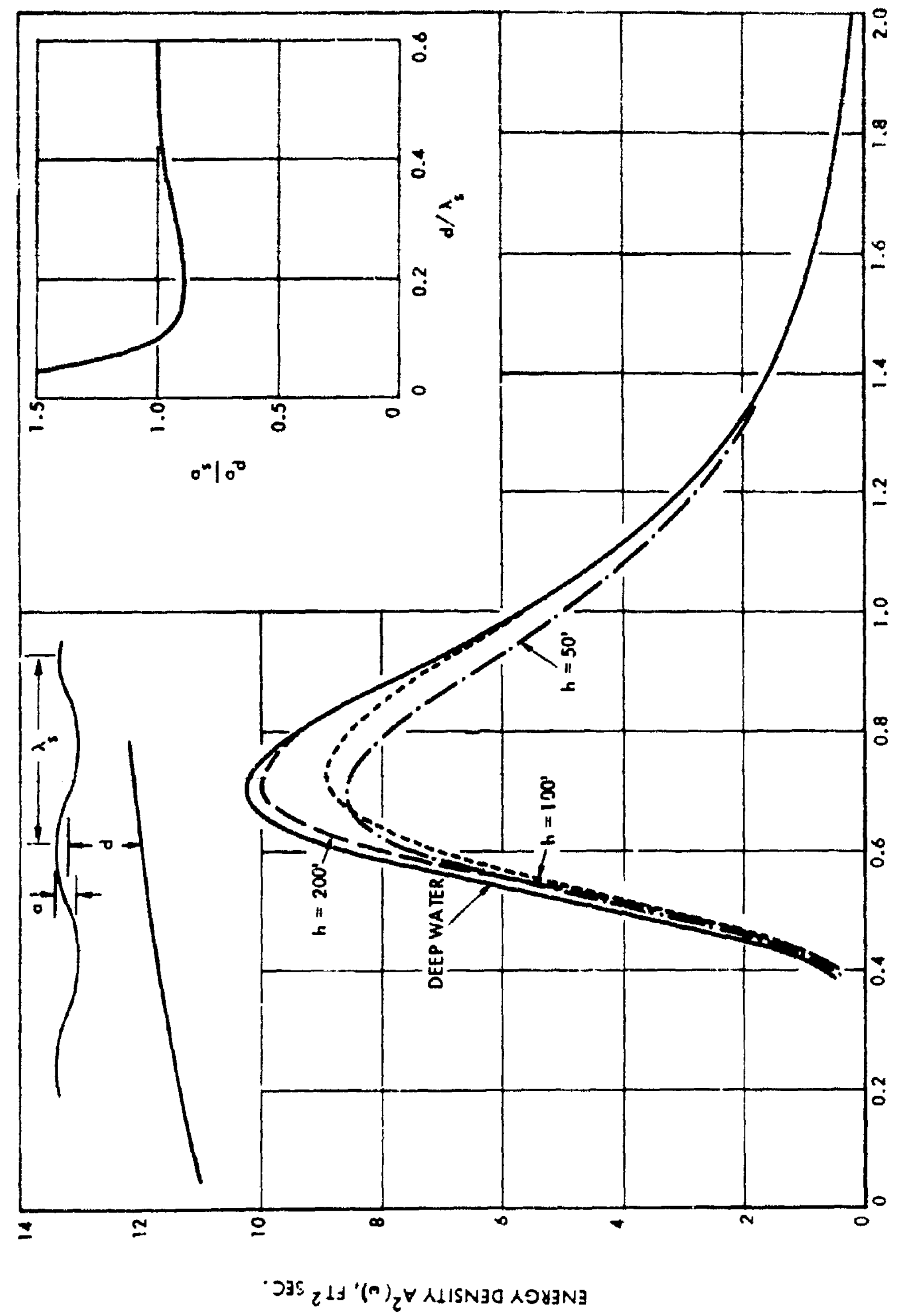


FIGURE 13 - NEUMANN'S SPECTRAL ENERGY DENSITY MODIFIED BY SHALLOW WATER EFFECT

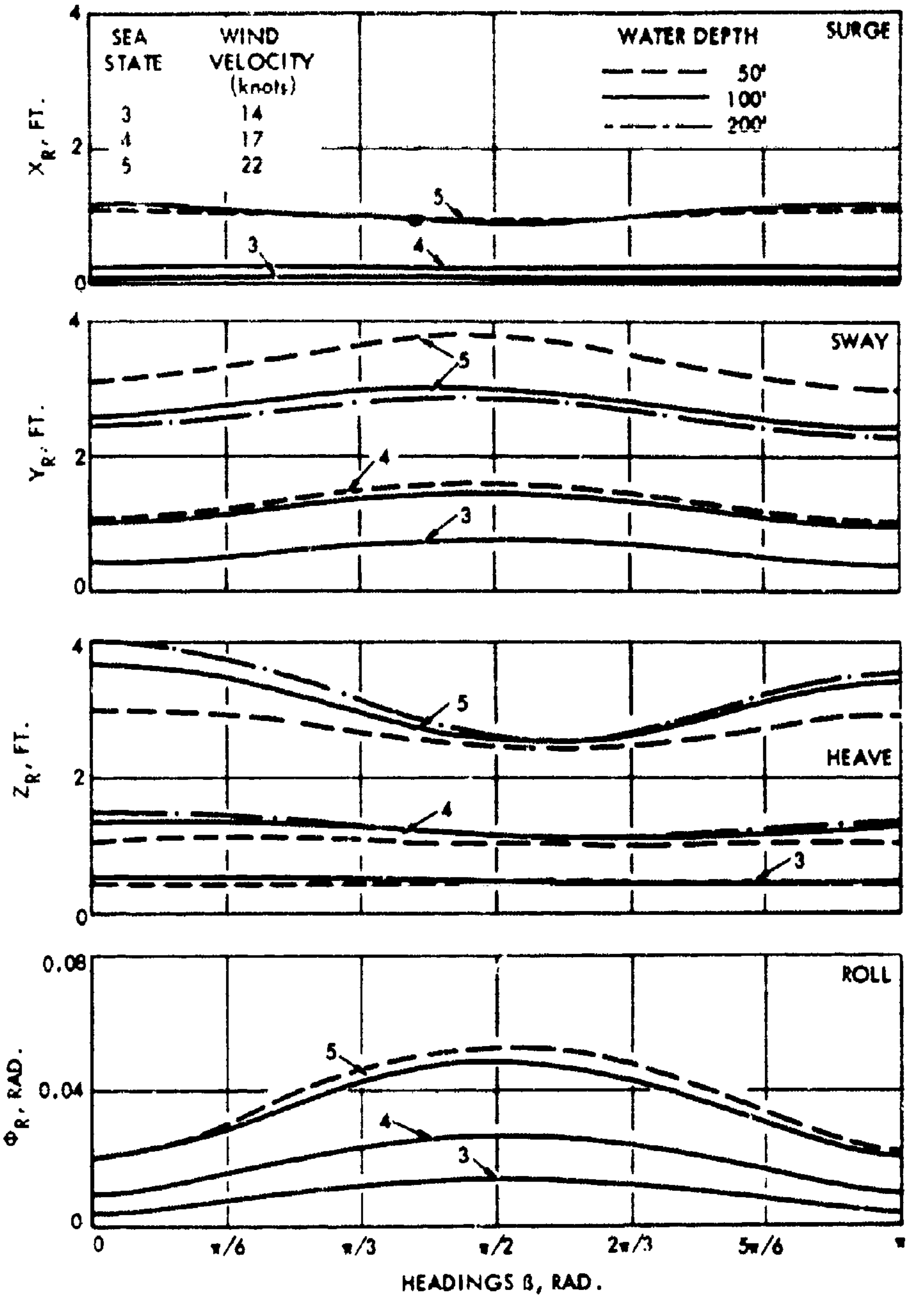
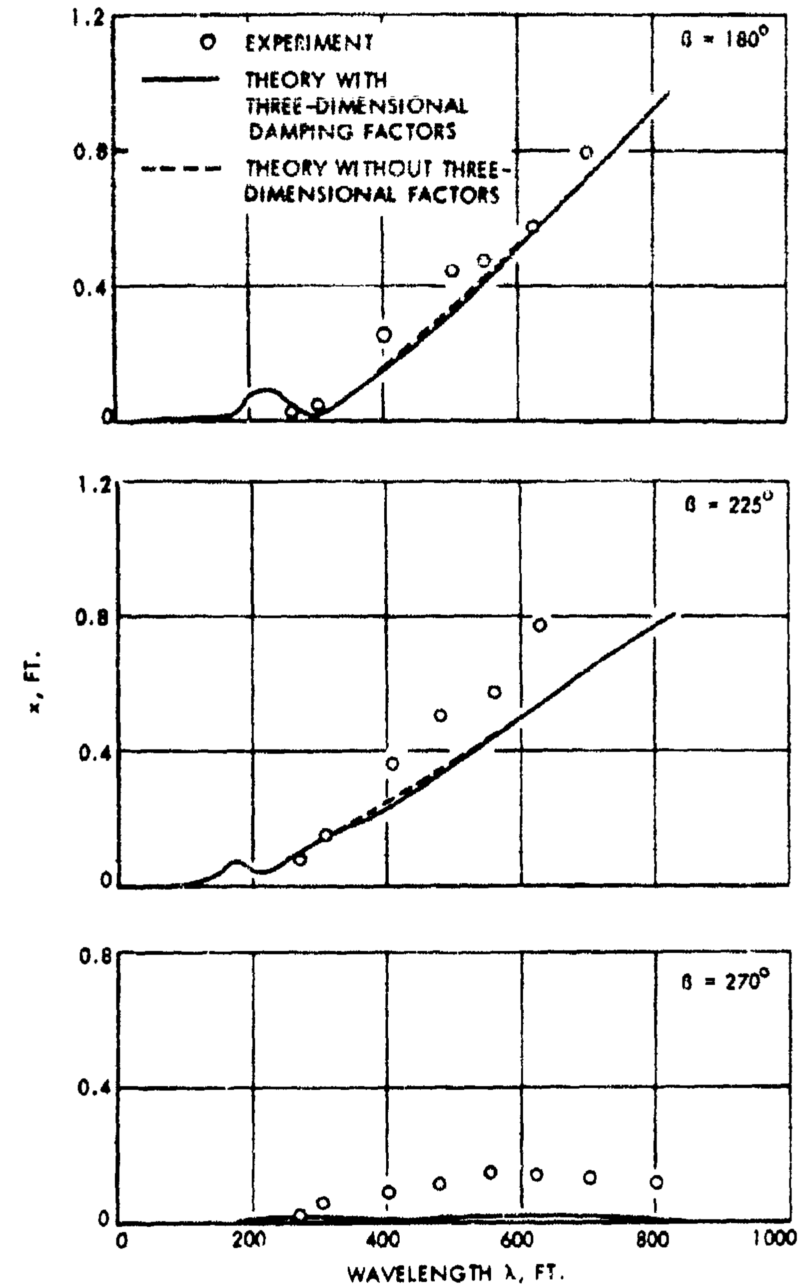
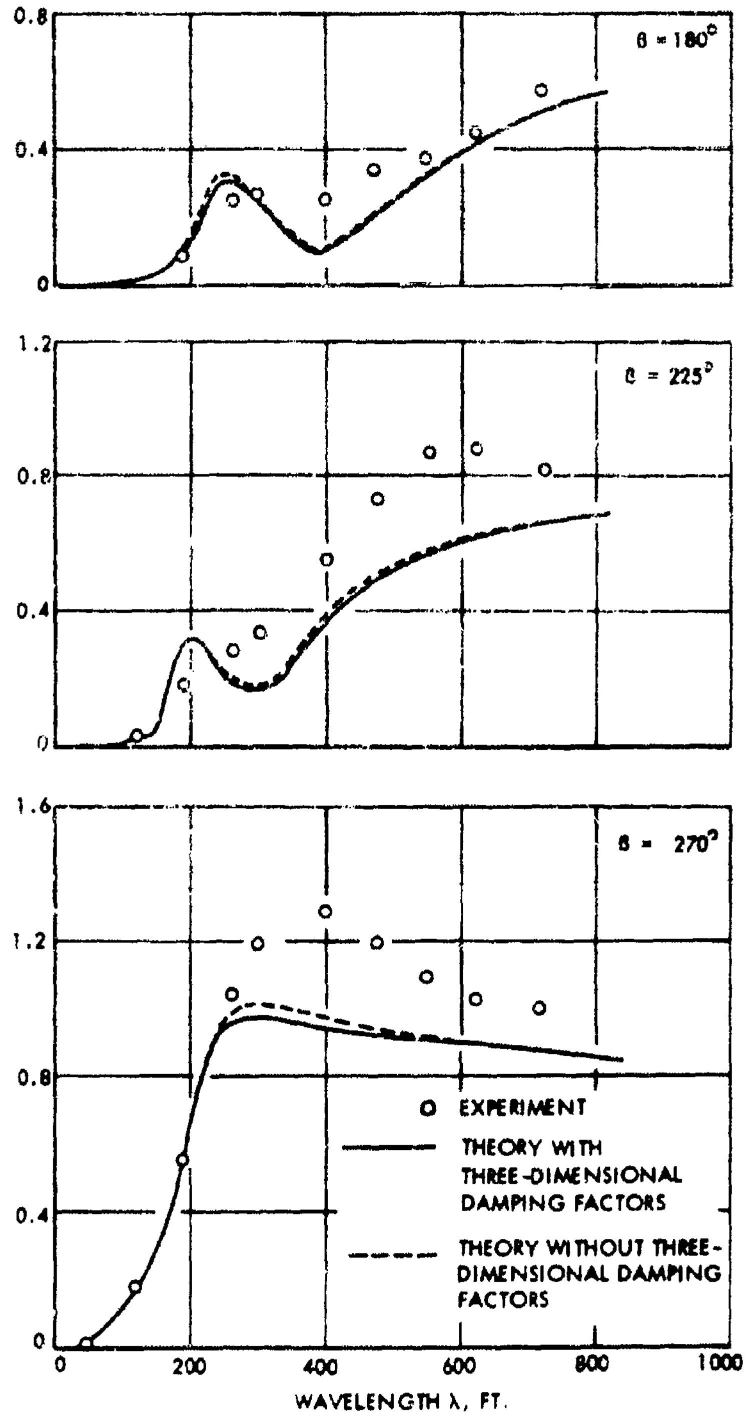


FIGURE 14 - R.M.S. VALUES OF RELATIVE MOTIONS BETWEEN COMET AND PAGE AT SHALLOW WATER IN NON-UNIDIRECTIONAL SEA



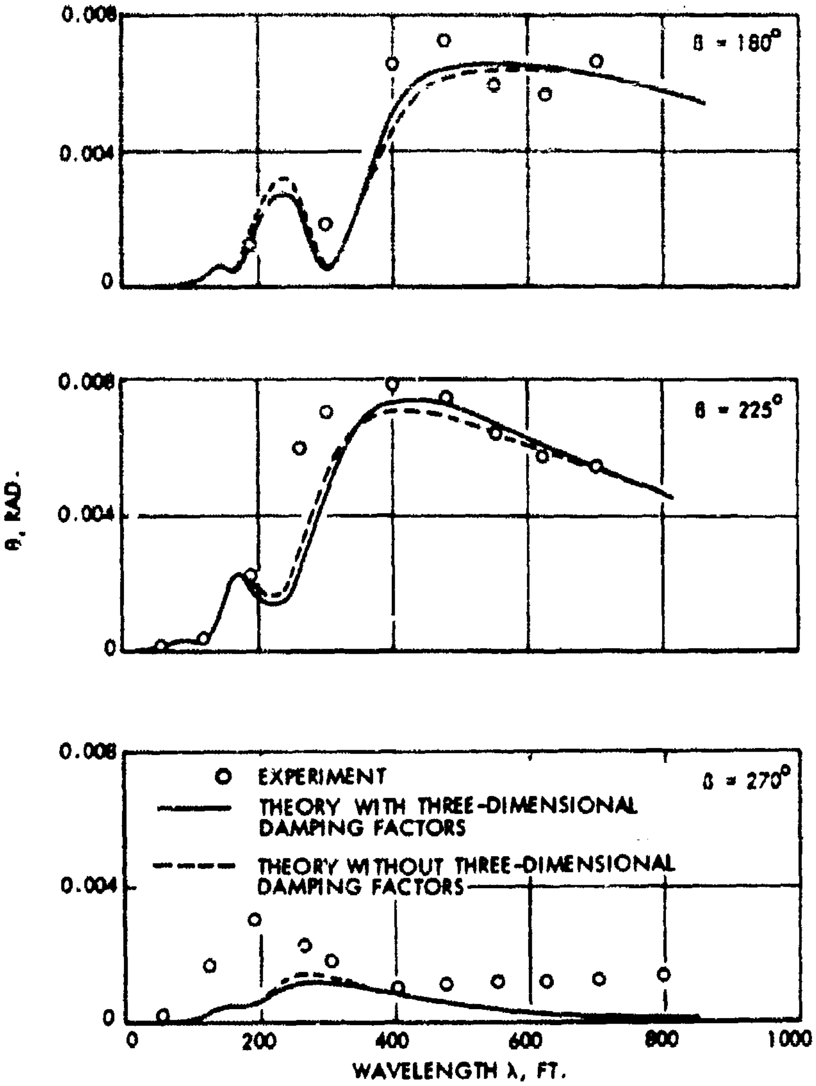
(a) AMPLITUDE OF SURGE

FIGURE 15 - COMPARISON BETWEEN THEORETICAL AND EXPERIMENTAL RESPONSES DUE TO UNIT-AMPLITUDE, REGULAR WAVES FOR COMET AT WATER DEPTH OF 100 FEET



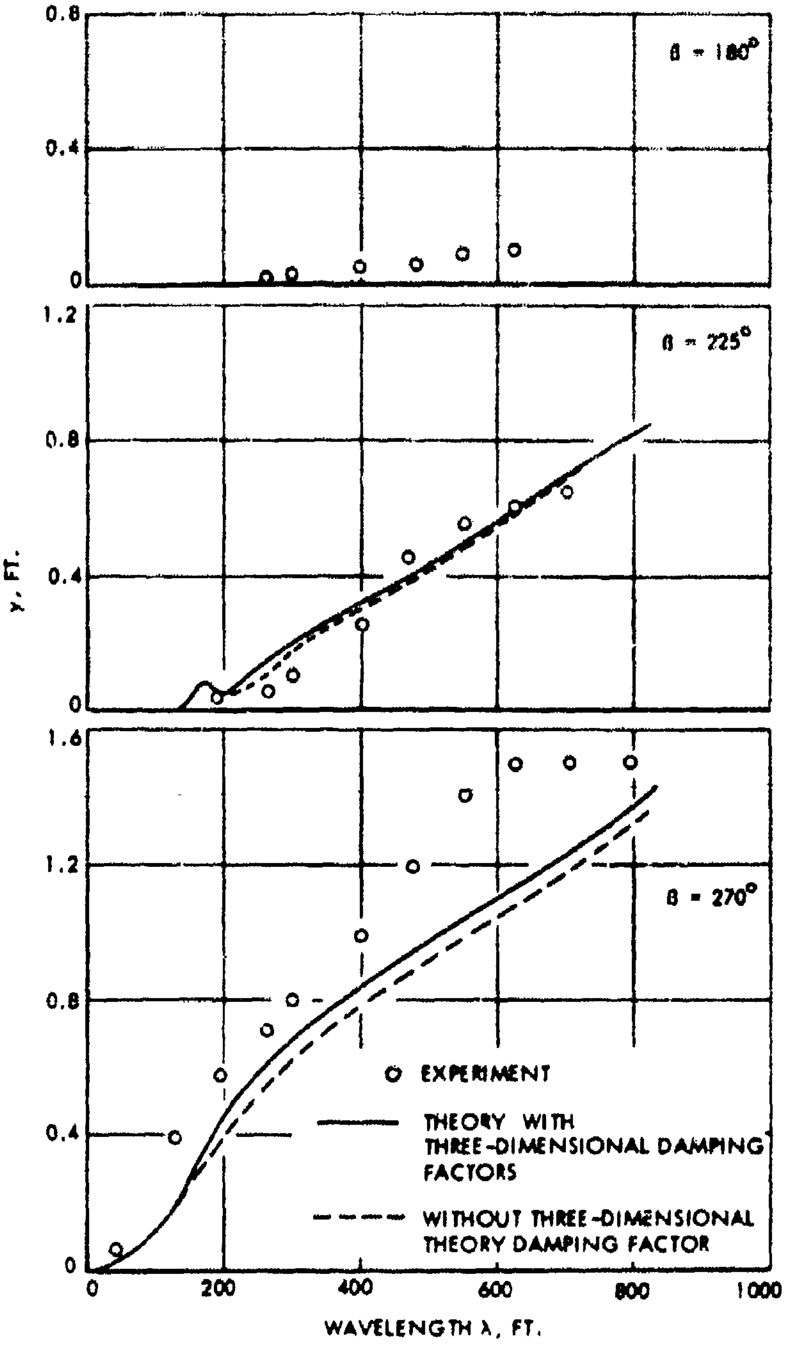
(b) AMPLITUDE OF HEAVE

FIGURE 15 - (CONTINUED)



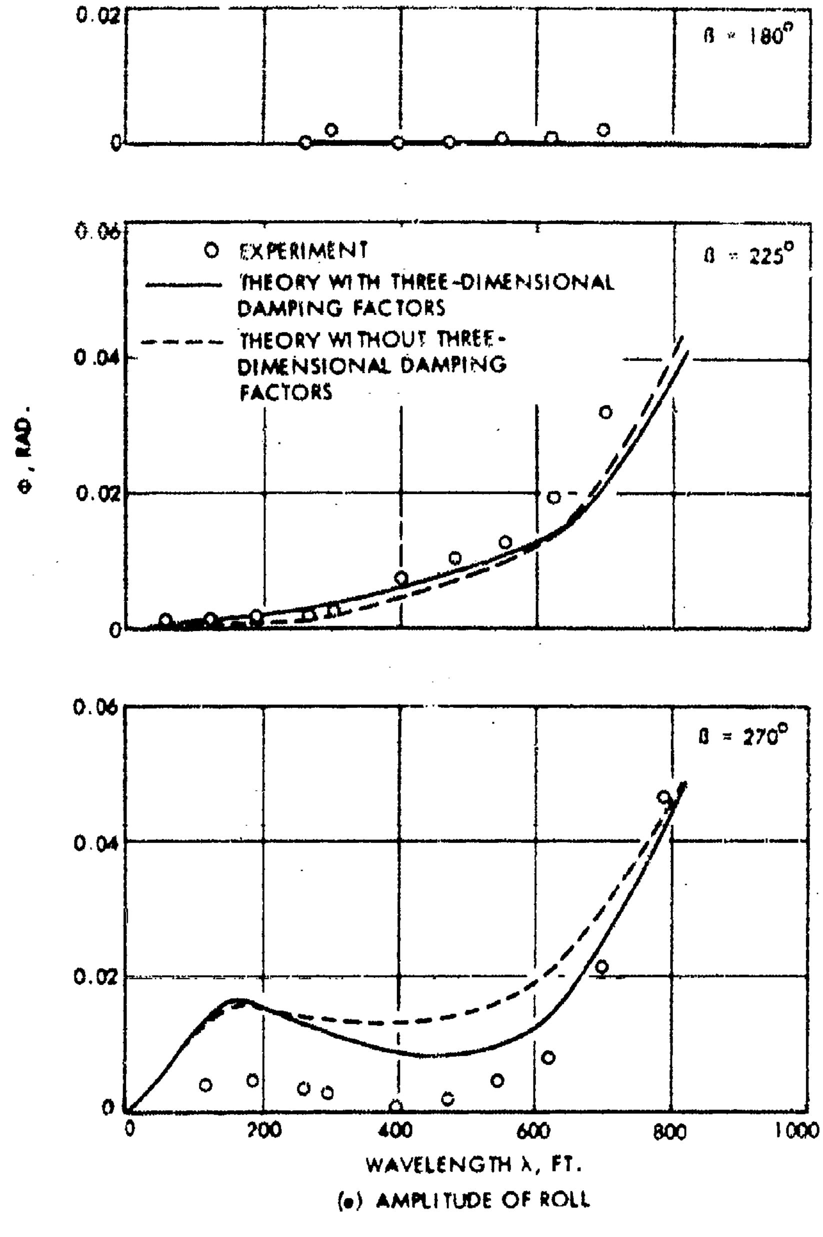
(c) AMPLITUDE OF PITCH

FIGURE 15 - (CONTINUED)



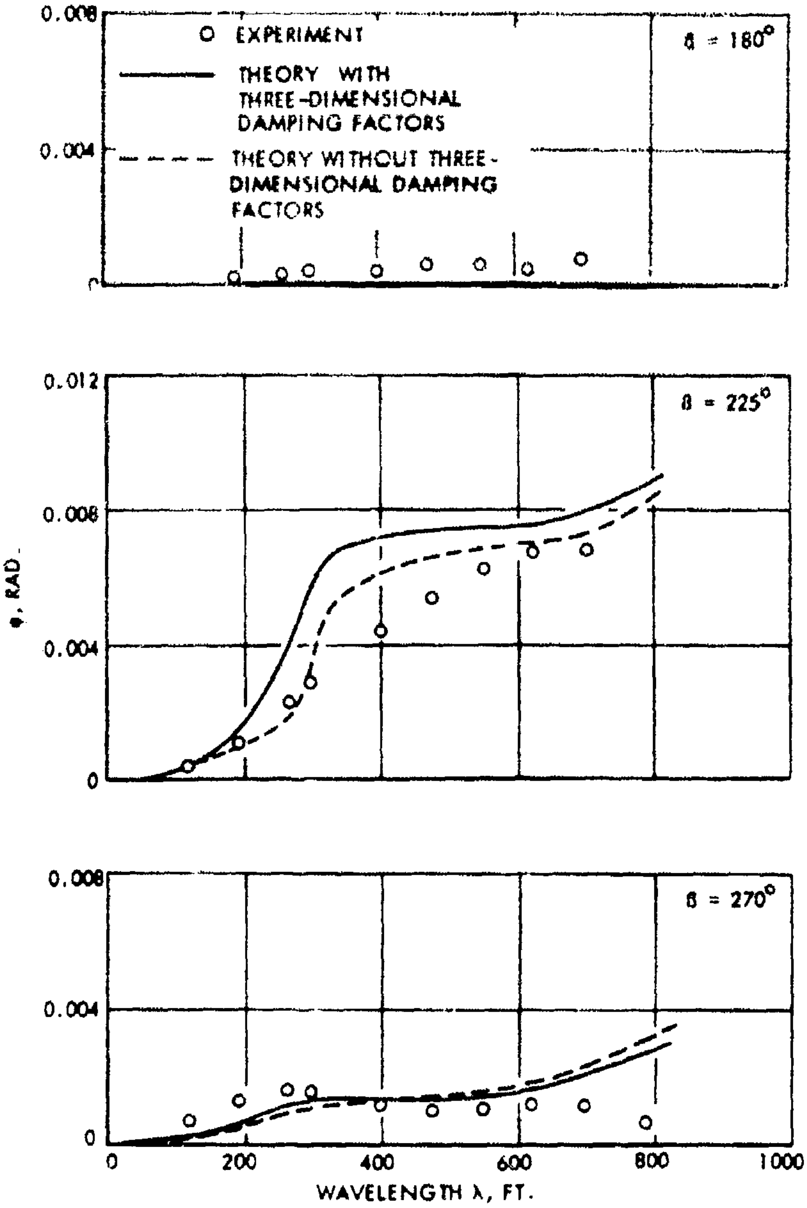
(d) AMPLITUDE OF SWAY

FIGURE 15 - (CONTINUED)



(e) AMPLITUDE OF ROLL

FIGURE 15 - (CONTINUED)



(f) AMPLITUDE OF YAW

FIGURE 15 - (CONTINUED)

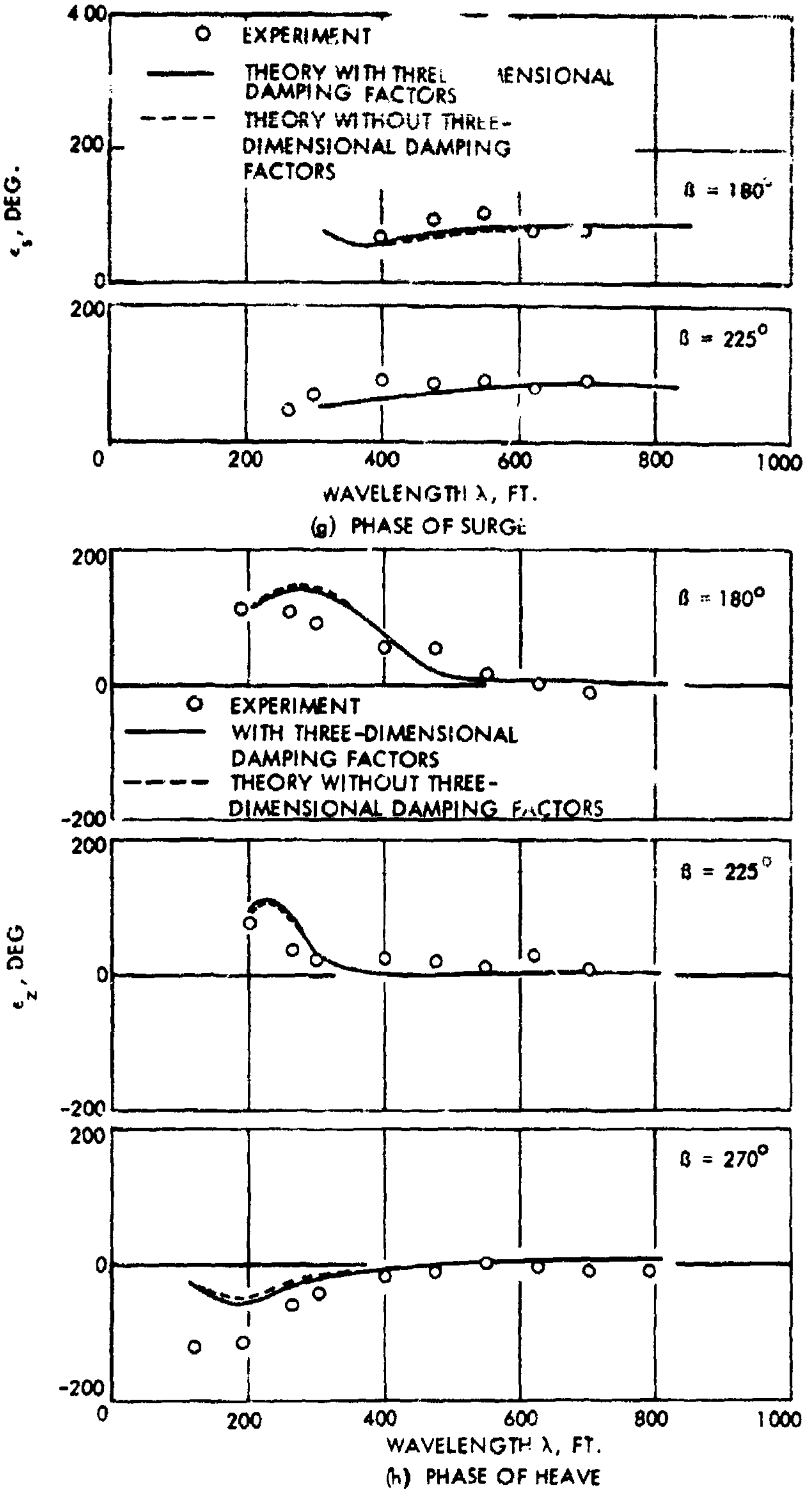
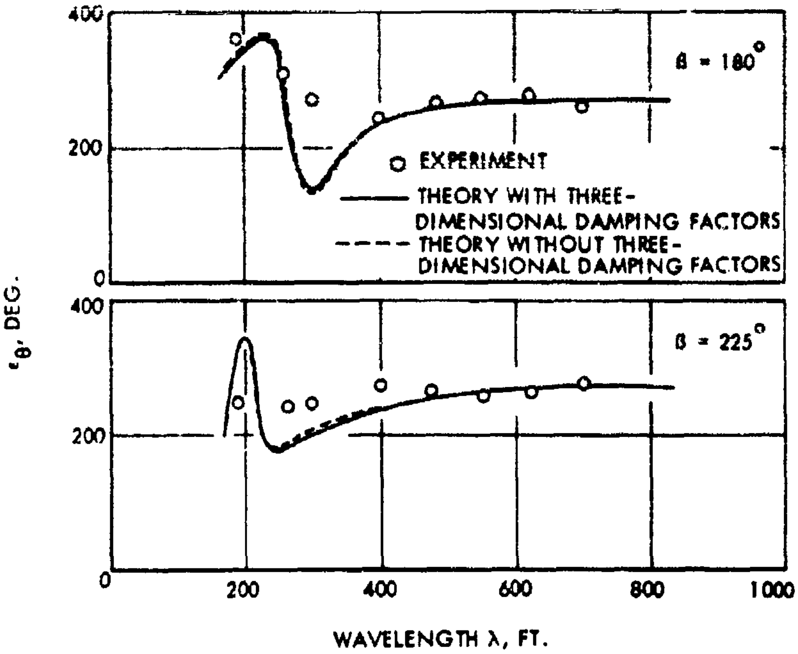
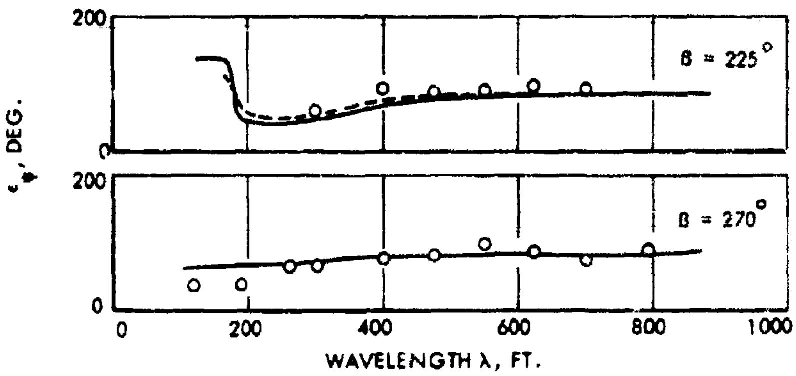


FIGURE 15 - (CONTINUED)

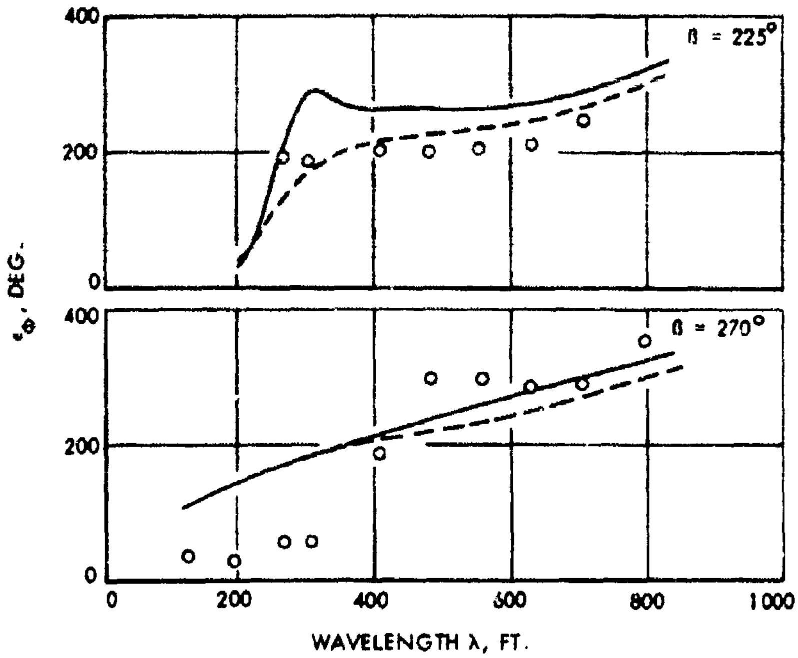


(i) PHASE OF PITCH

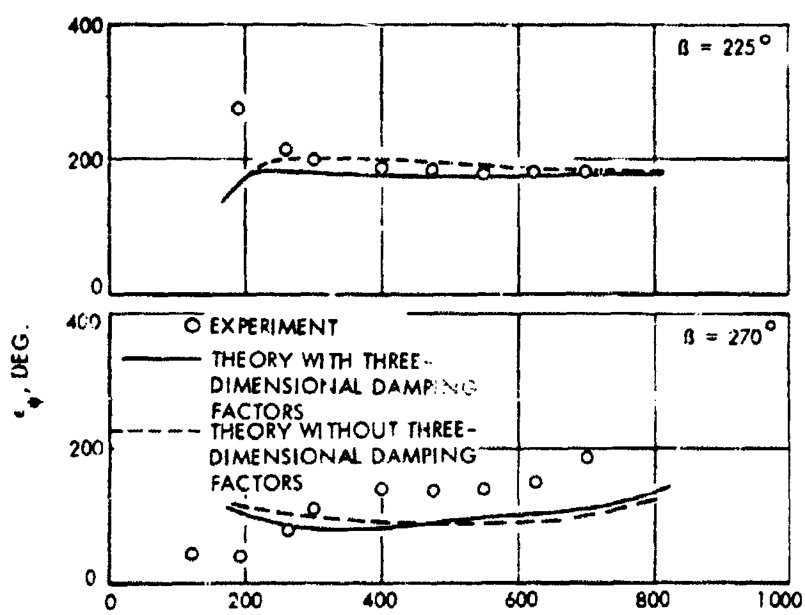


(j) PHASE OF SWAY

FIGURE 15 - (CONTINUED)

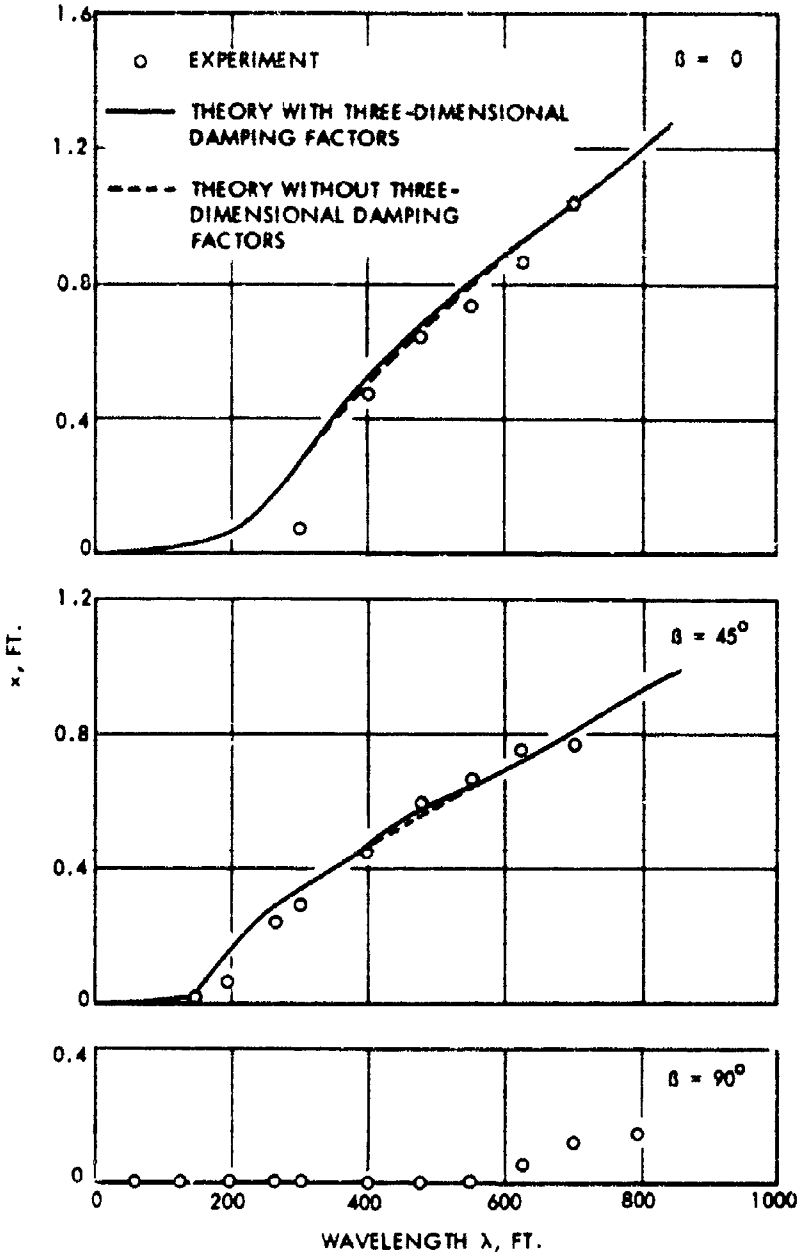


(k) PHASE OF ROLL



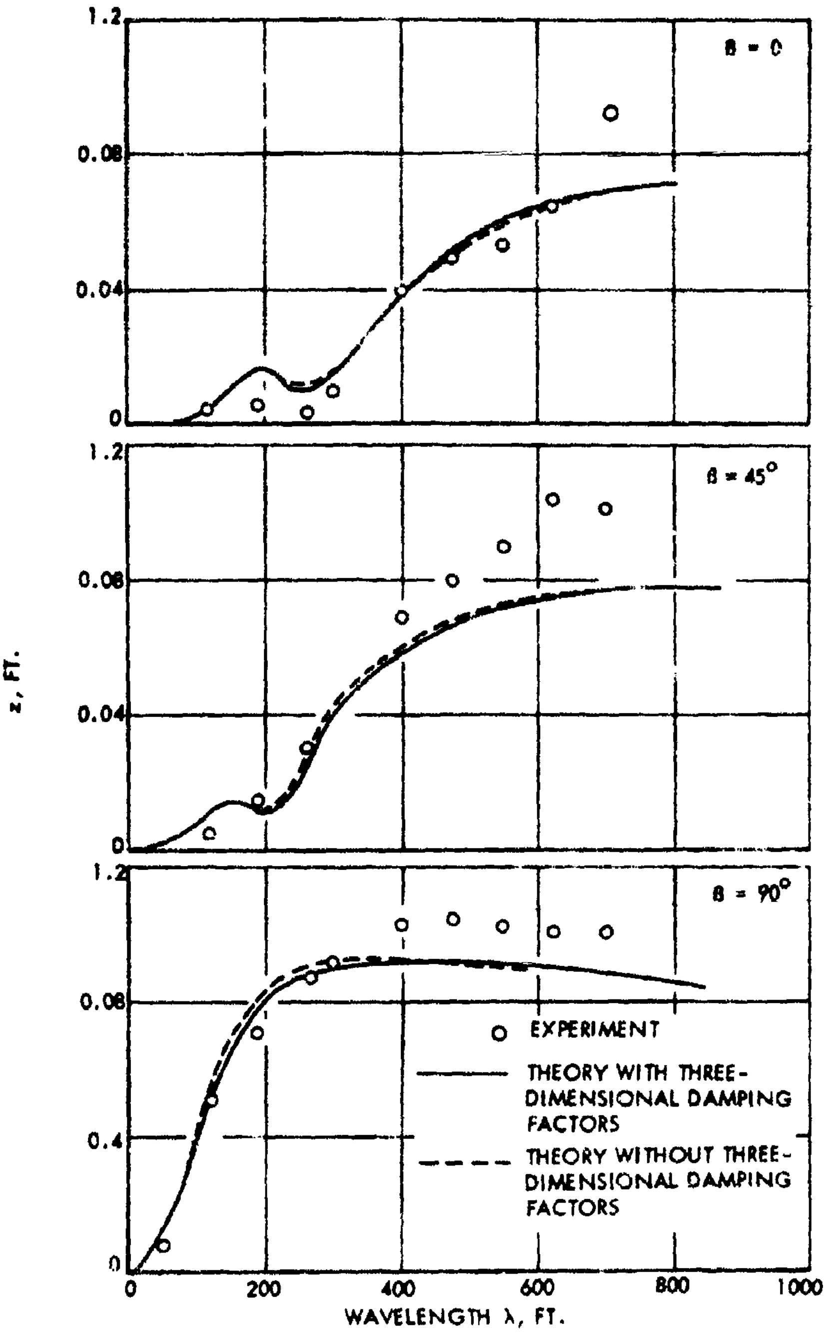
(ℓ) PHASE OF YAW

FIGURE 15 - (CONCLUDED)



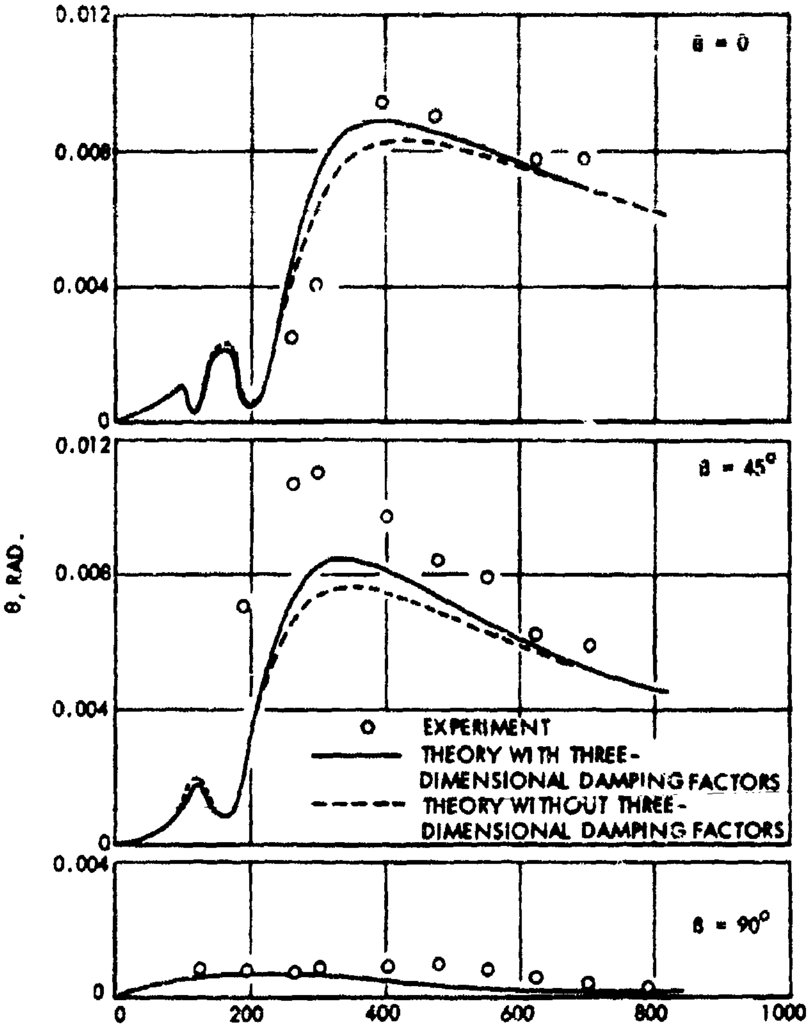
(a) AMPLITUDE OF SURGE

FIGURE 16 - COMPARISON BETWEEN THEORETICAL AND EXPERIMENTAL RESPONSES DUE TO UNIT-AMPLITUDE, REGULAR WAVES FOR PAGE AT WATER DEPTH OF 100 FEET



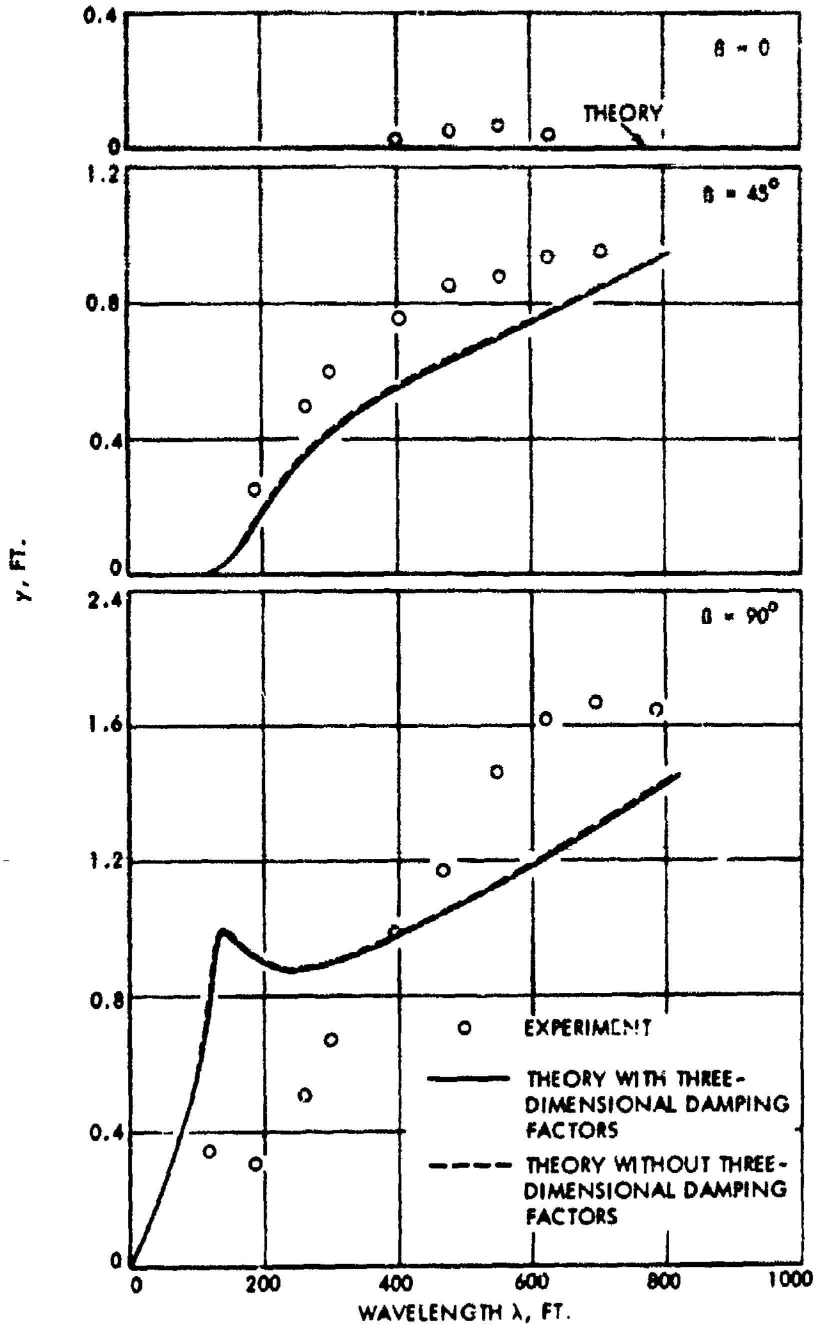
(b) AMPLITUDE OF HEAVE

FIGURE 16 - (CONTINUED)



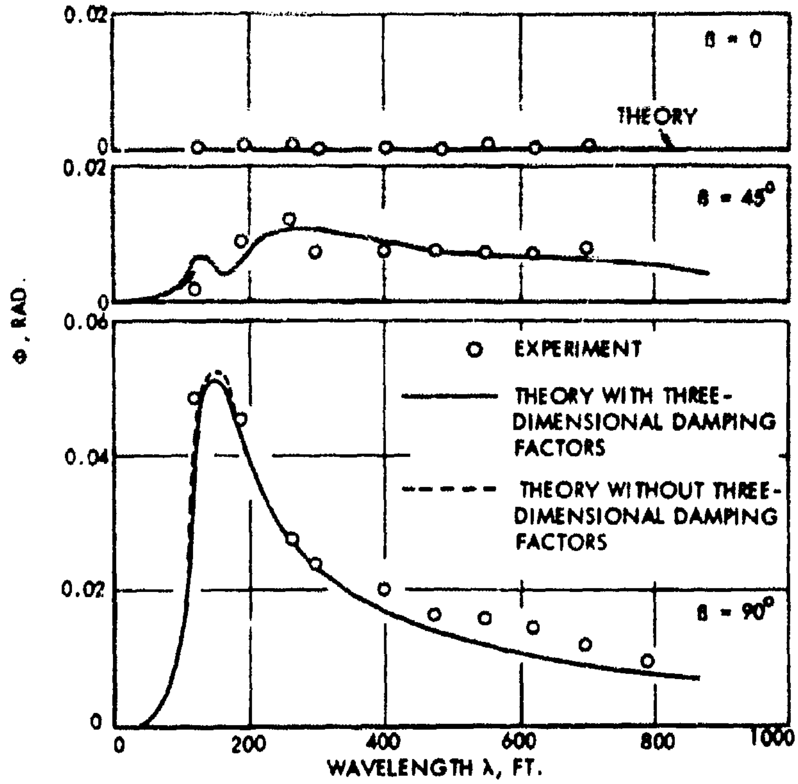
(c) AMPLITUDE OF PITCH

FIGURE 16 - (CONTINUED)

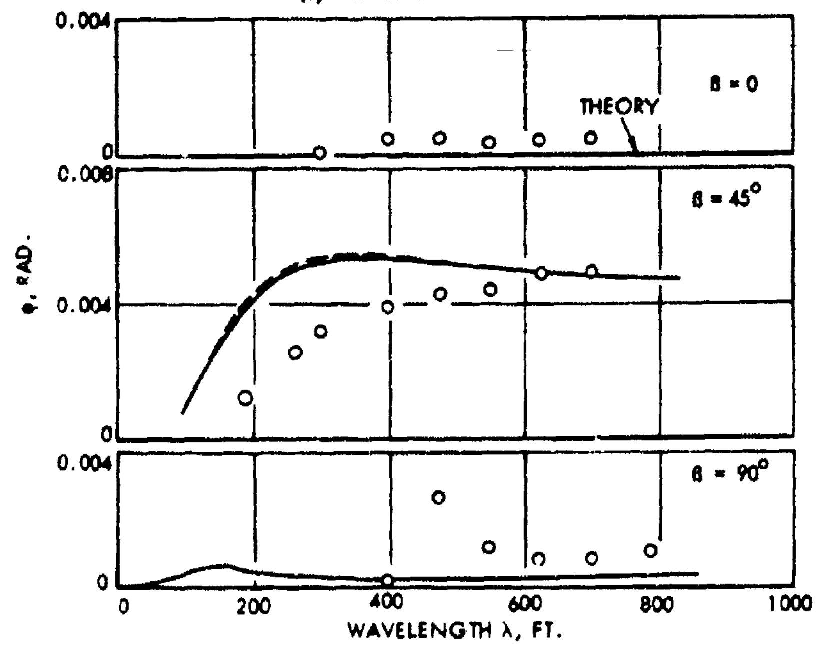


(d) AMPLITUDE OF SWAY

FIGURE 16 - (CONTINUED)

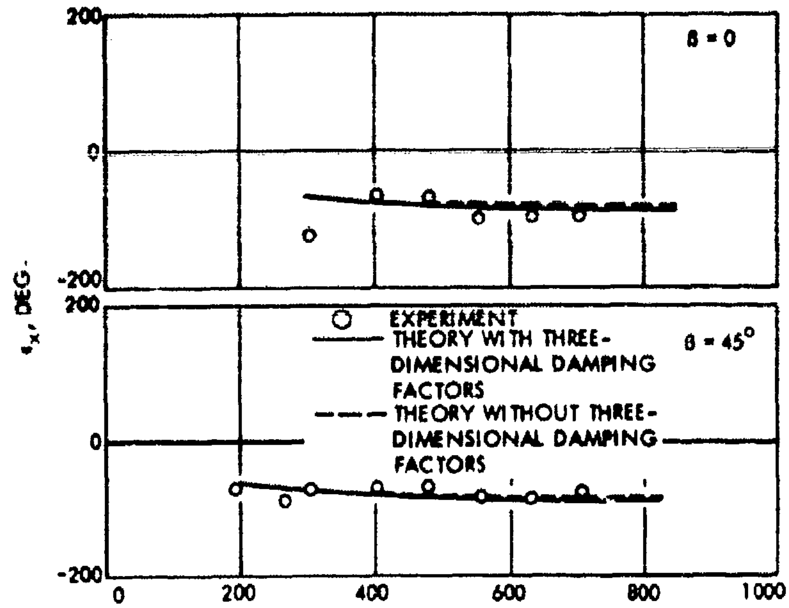


(e) AMPLITUDE OF ROLL

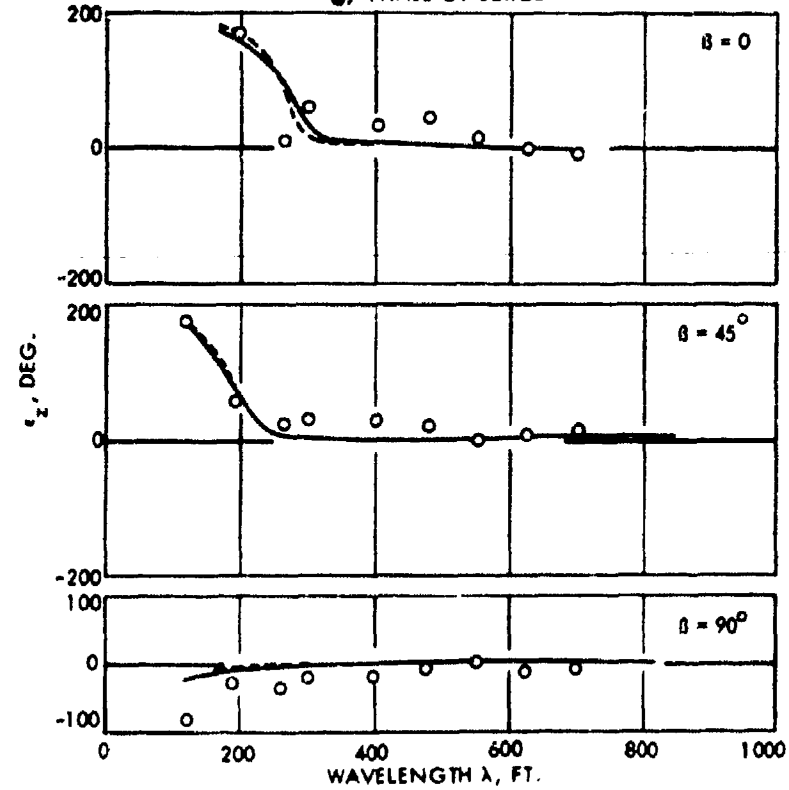


(f) AMPLITUDE OF YAW

FIGURE 16 - (CONTINUED)

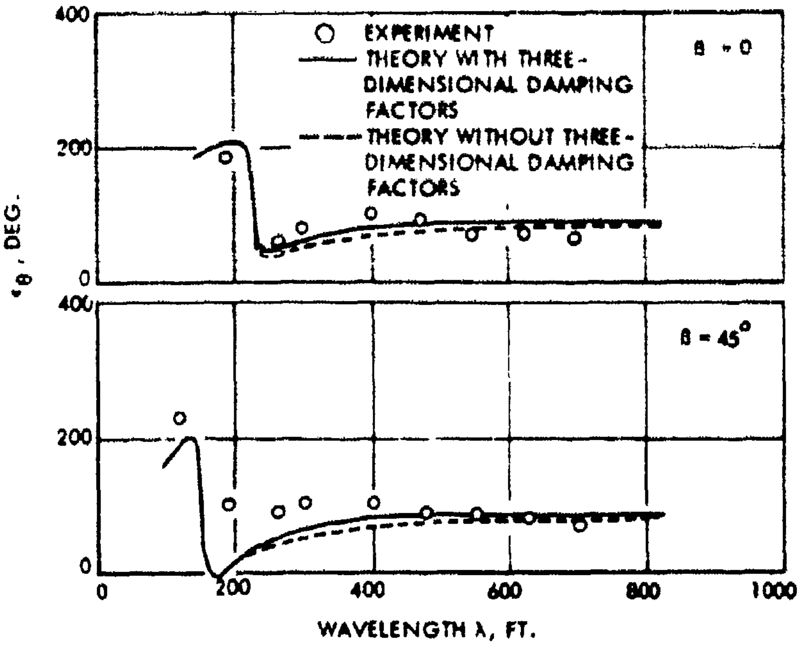


(g) PHASE OF SURGE

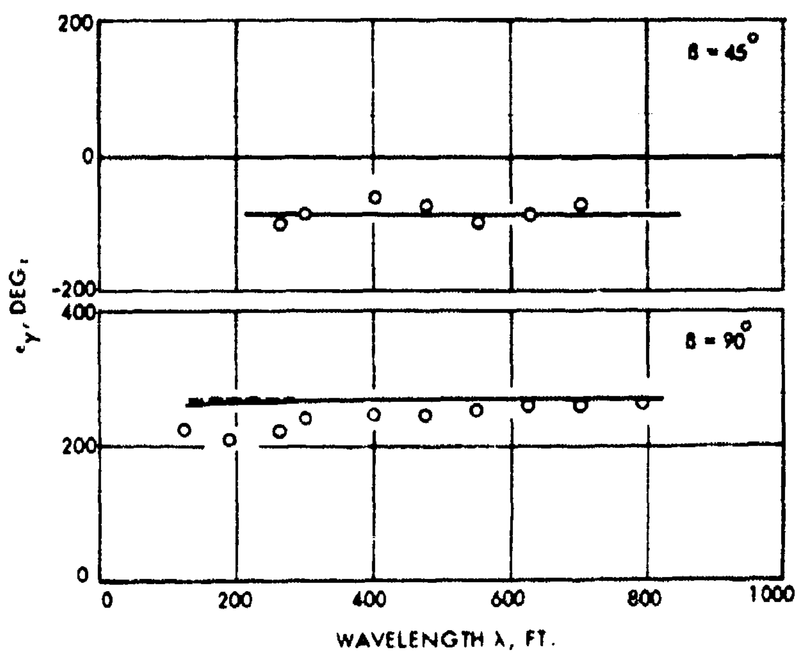


(h) PHASE OF HEAVE

FIGURE 16 - (CONTINUED)

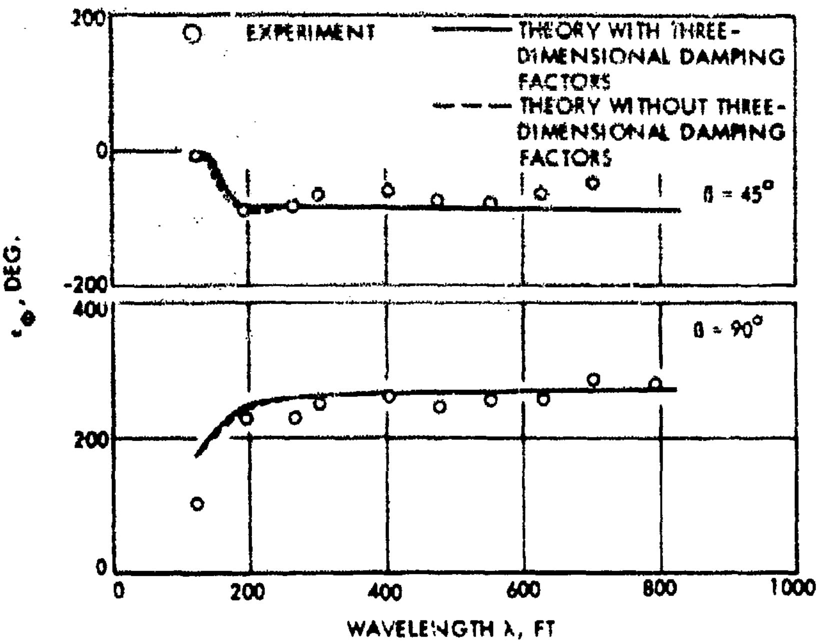


(i) PHASE OF PITCH

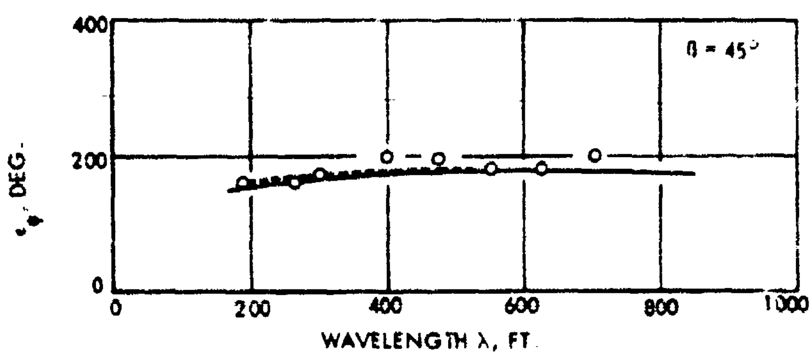


(j) PHASE OF SWAY

FIGURE 16 - (CONTINUED)



(k) PHASE OF ROLL



(ℓ) PHASE OF YAW

FIGURE 16 - (CONCLUDED)

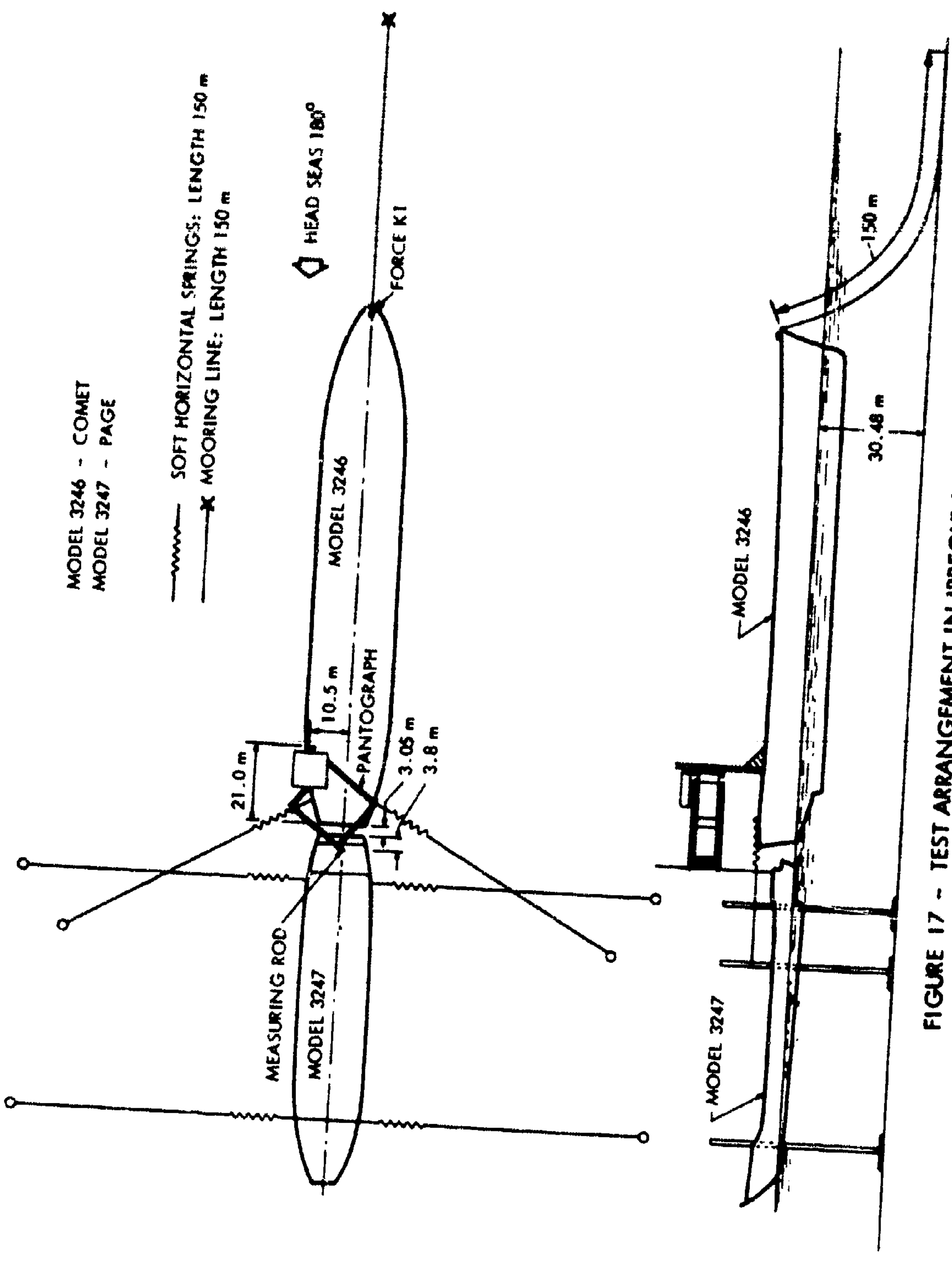


FIGURE 17 - TEST ARRANGEMENT IN IRREGULAR WAVES, HEAD SEA

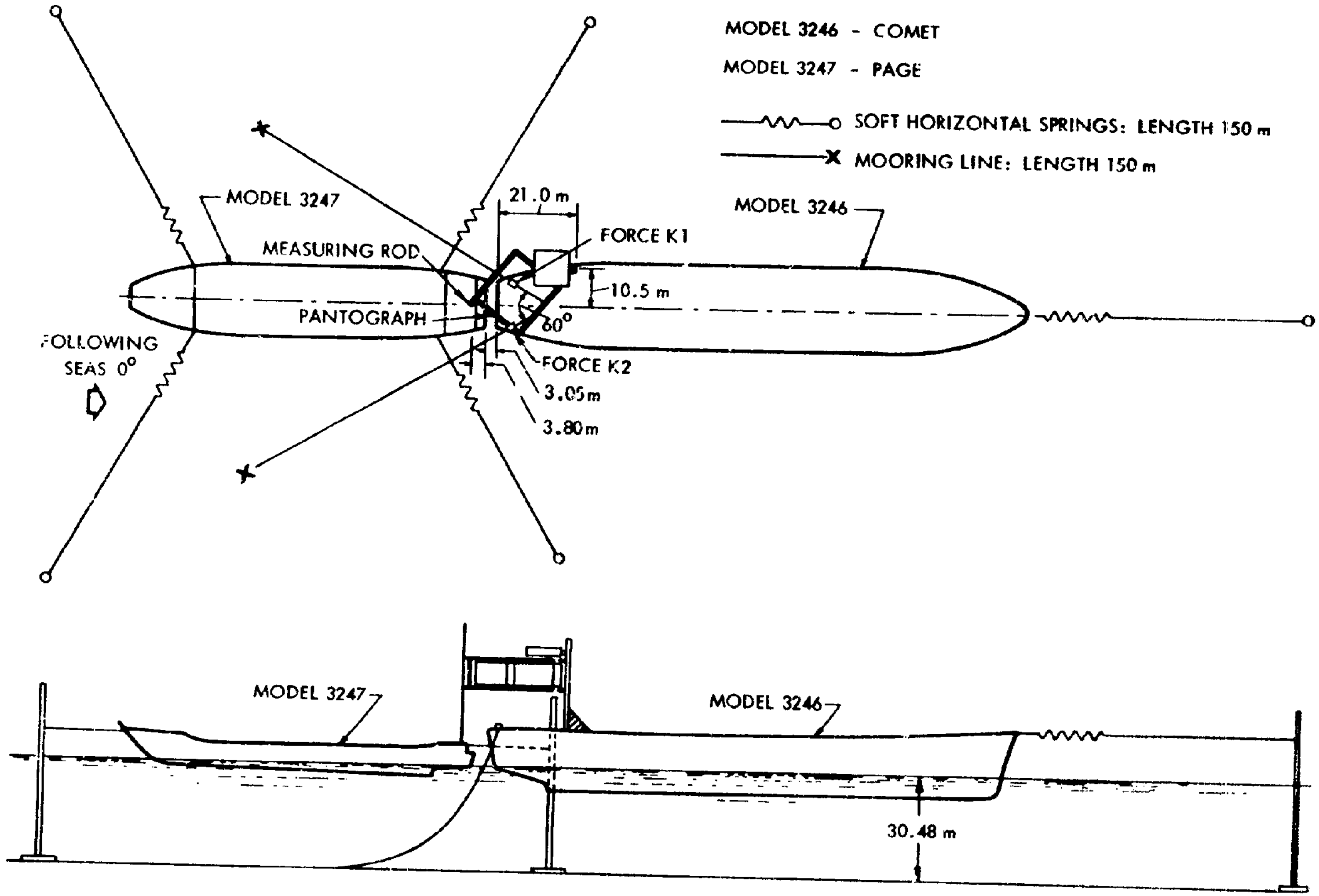
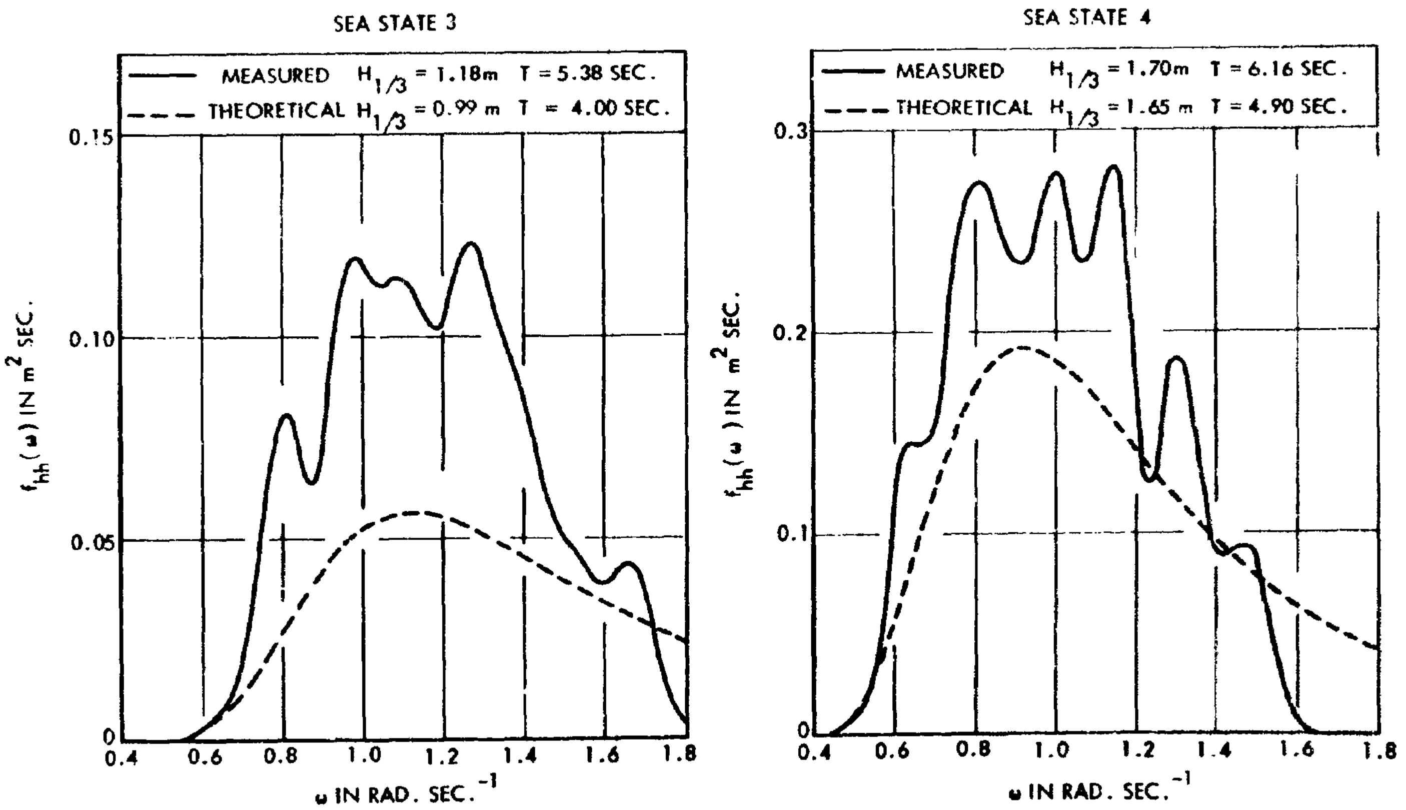


FIGURE 18 - TEST ARRANGEMENT IN IRREGULAR WAVES, FOLLOWING SEA

WAVE SPECTRA



NOTE: $H_{1/3} = 4\left[\int_o^\infty f_{hh}\, dw\right]^{1/2}$ $\quad T = \dfrac{2\pi \int_o^\infty f_{hh}\, dw}{\int_o^\infty \omega f_{hh}\, dw}$

FIGURE 19 - MEASURED WAVE SPECTRA IN IRREGULAR WAVE TEST

WAVE SPECTRA

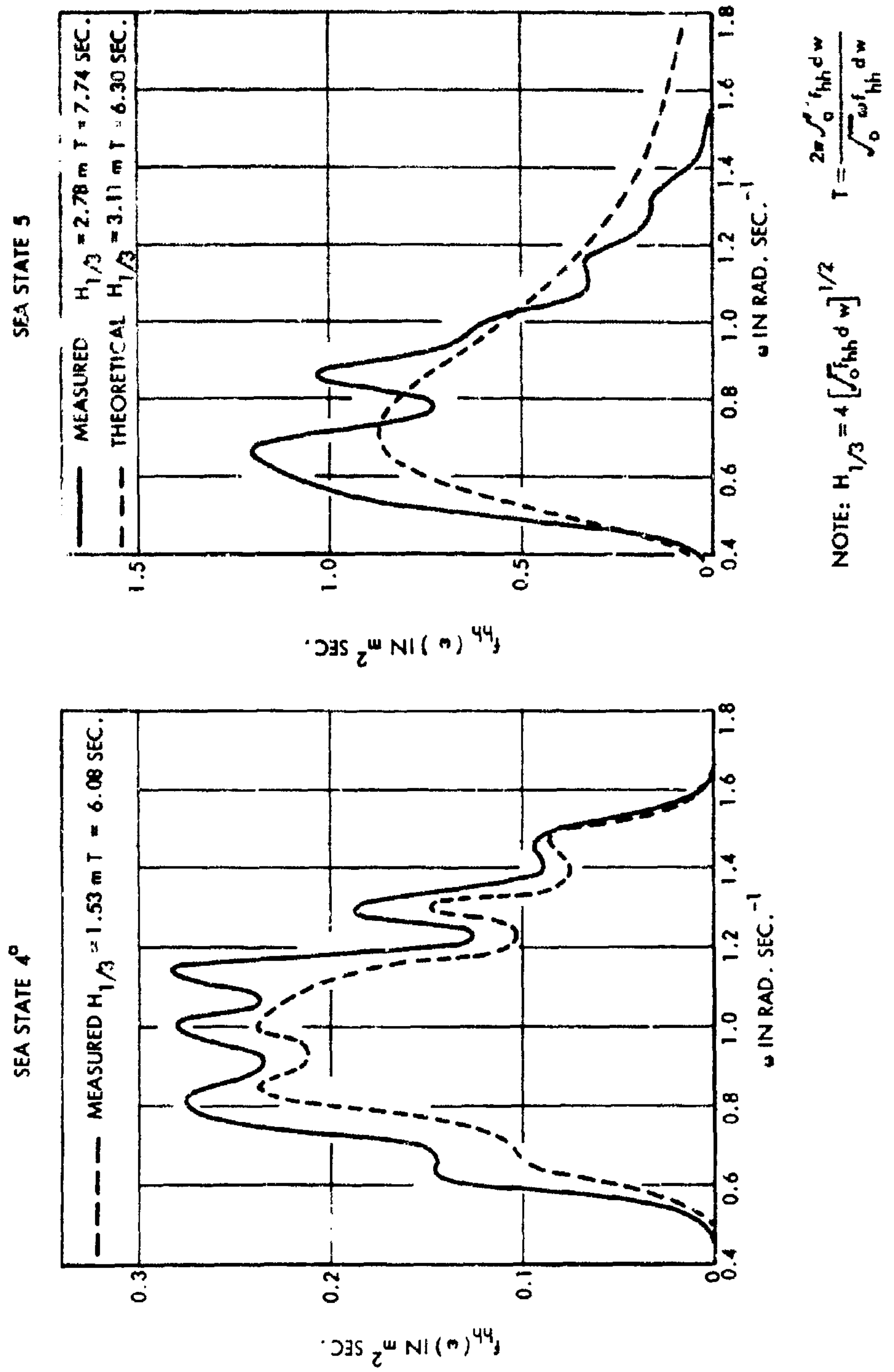


FIGURE 19 - (CONCLUDED)

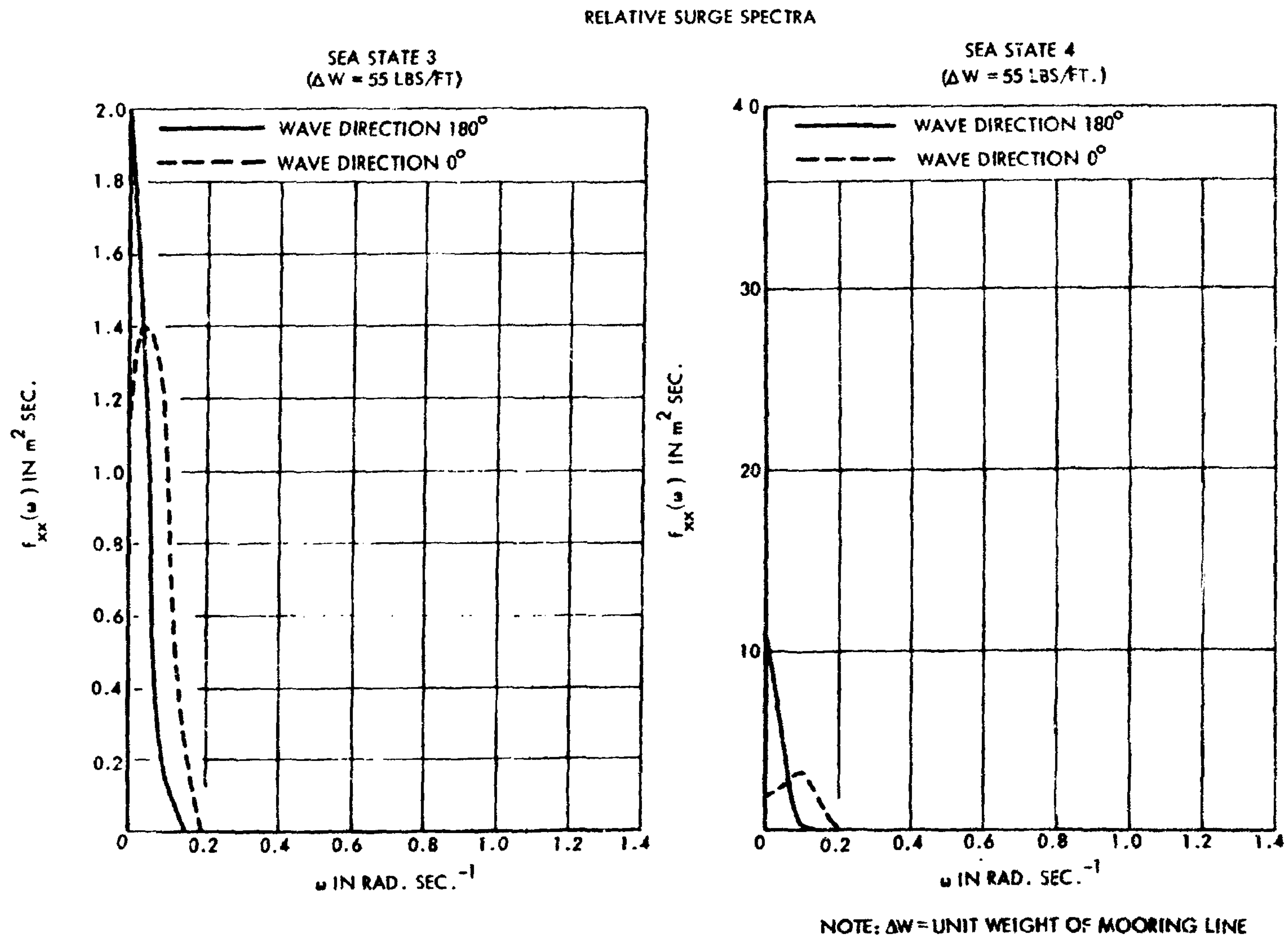


FIGURE 20 - MEASURED SPECTRA OF RELATIVE SURGE BETWEEN COMET AND PAGE IN IRREGULAR WAVE TEST

RELATIVE SURGE SPECTRA

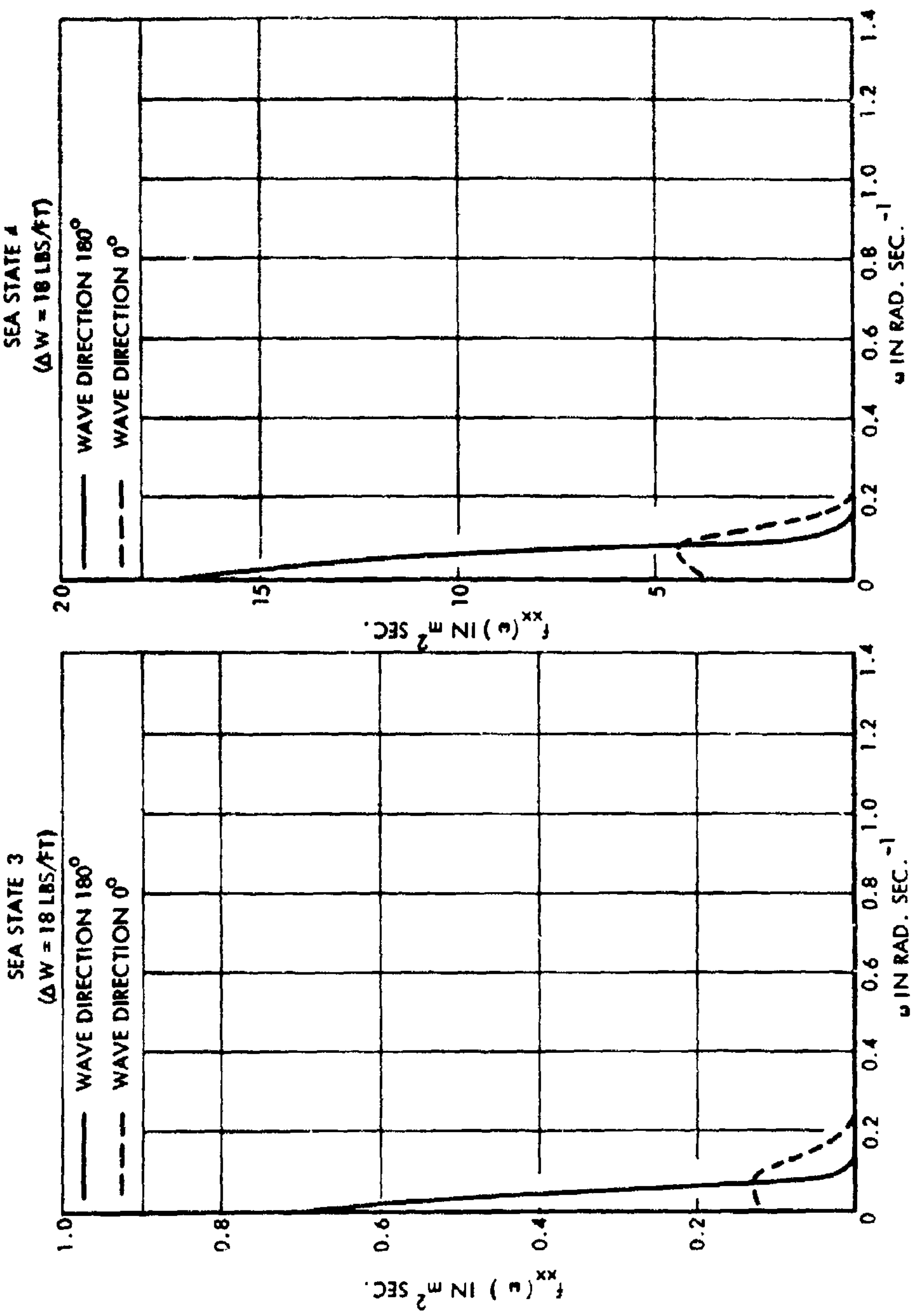


FIGURE 20 - (CONTINUED)

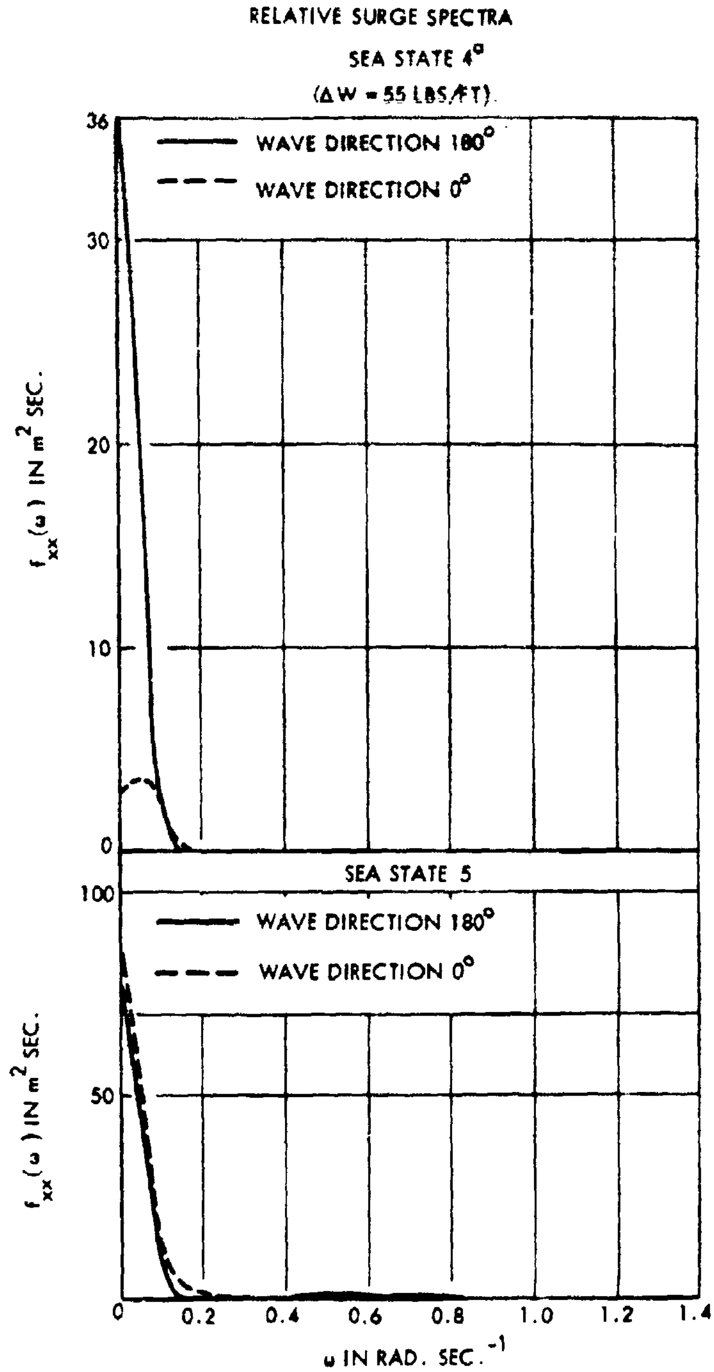


FIGURE 20 - (CONTINUED)

RELATIVE SURGE SPECTRA

SEA STATE 5

(ΔW = 18 LBS/FT)

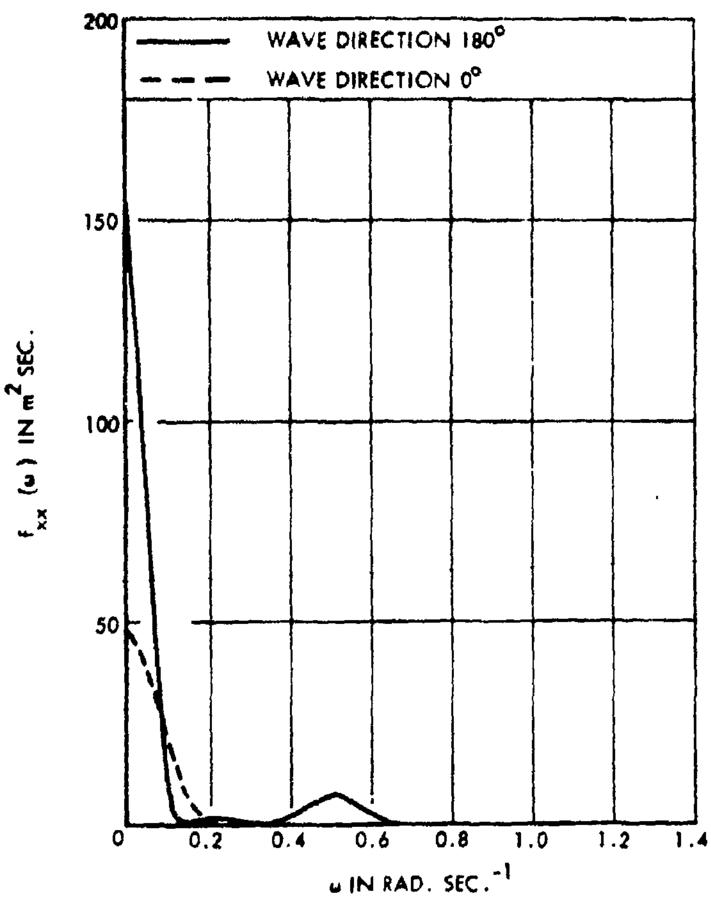
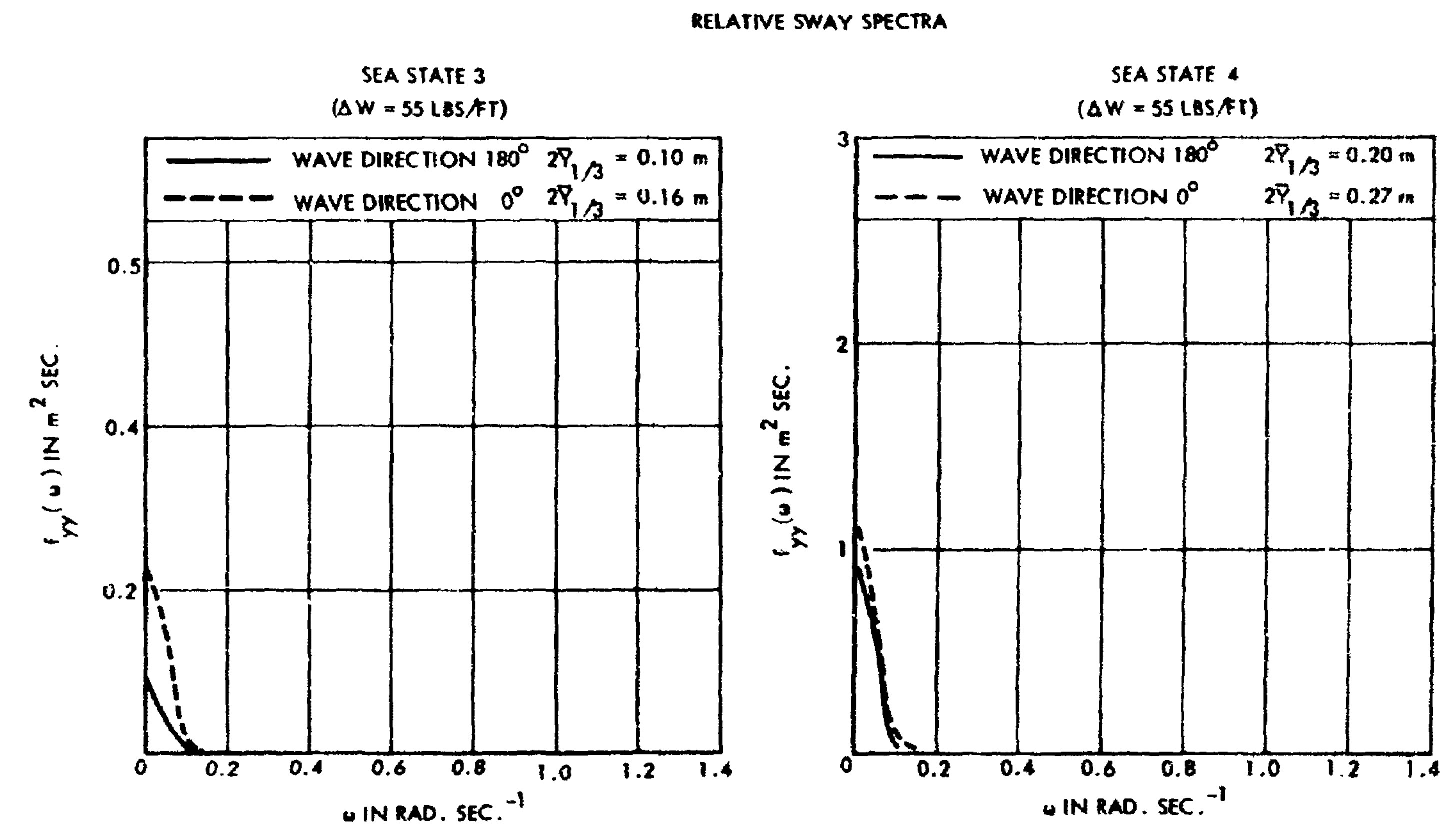


FIGURE 20 - (CONCLUDED)



NOTE: ΔW = UNIT WEIGHT OF MOORING LINE

FIGURE 21 – MEASURED SPECTRA OF RELATIVE SWAY BETWEEN COMET AND PAGE IN IRREGULAR WAVE TEST

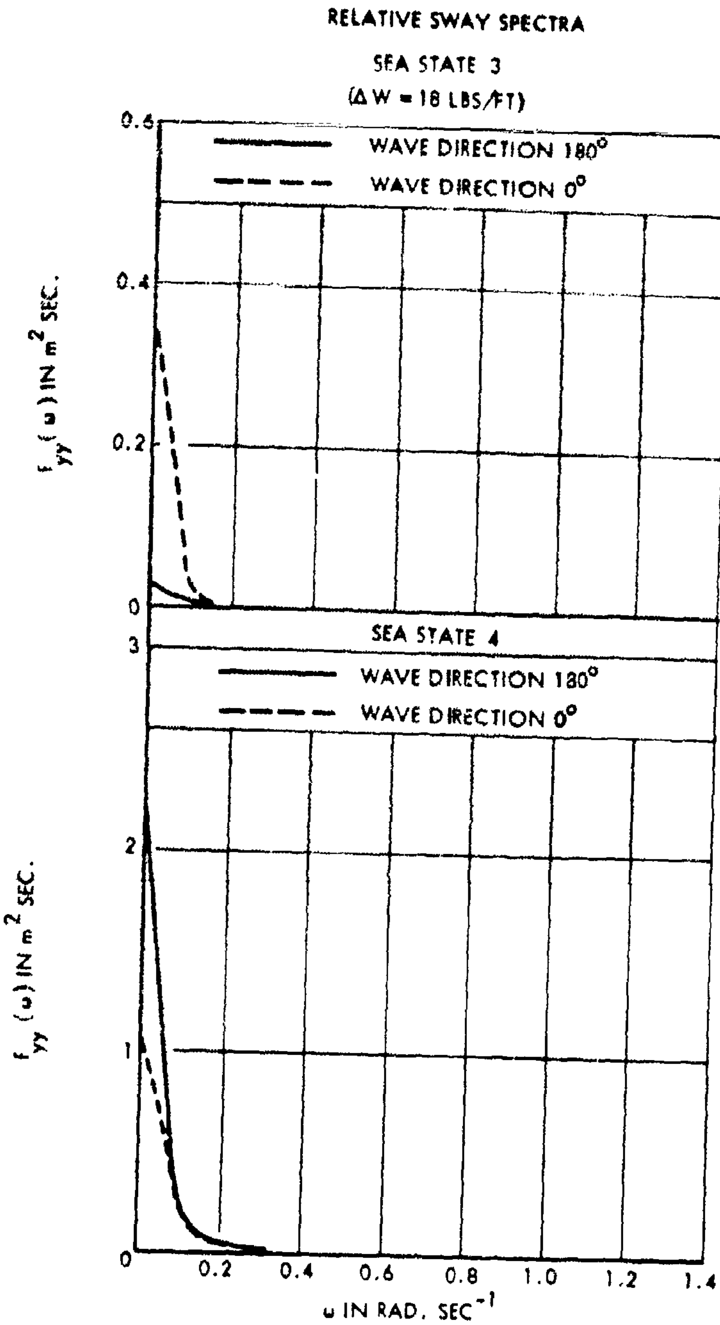


FIGURE 21 - (CONTINUED)

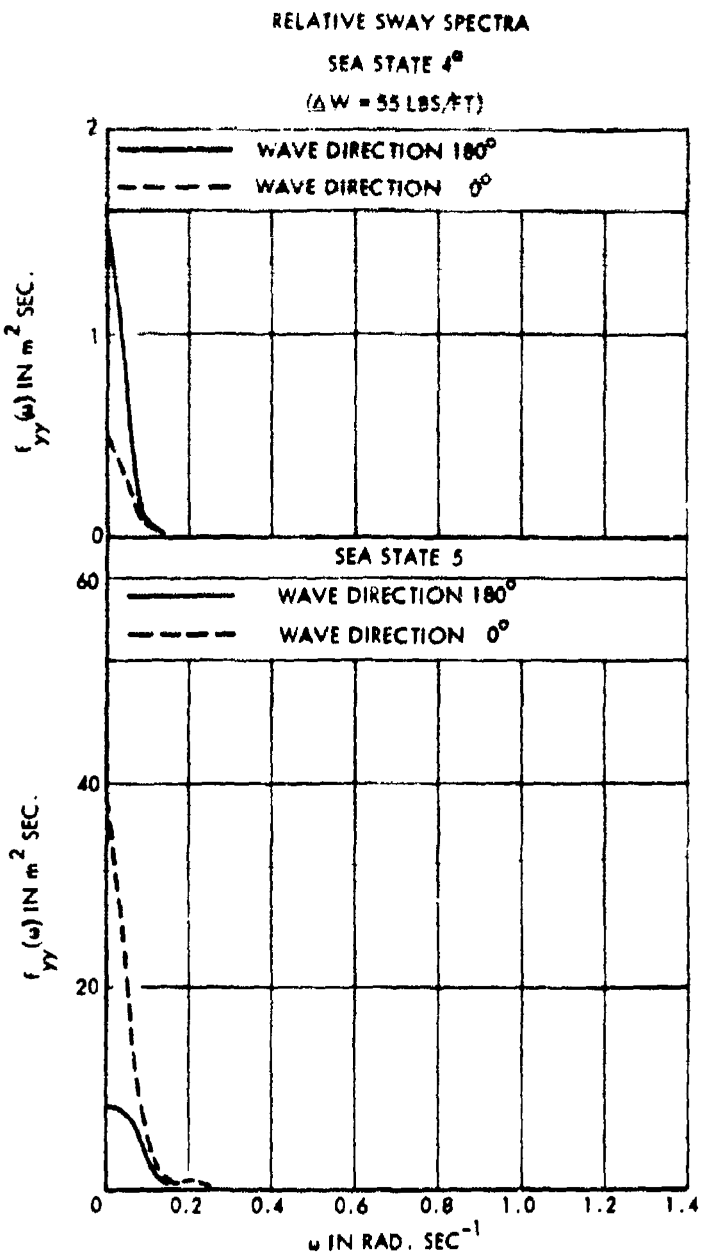


FIGURE 21 - (CONTINUED)

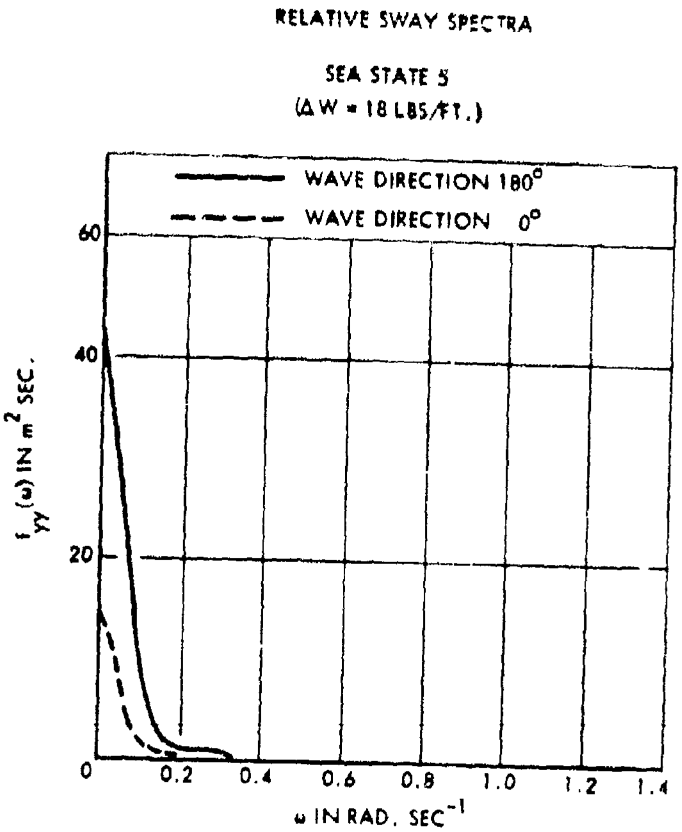
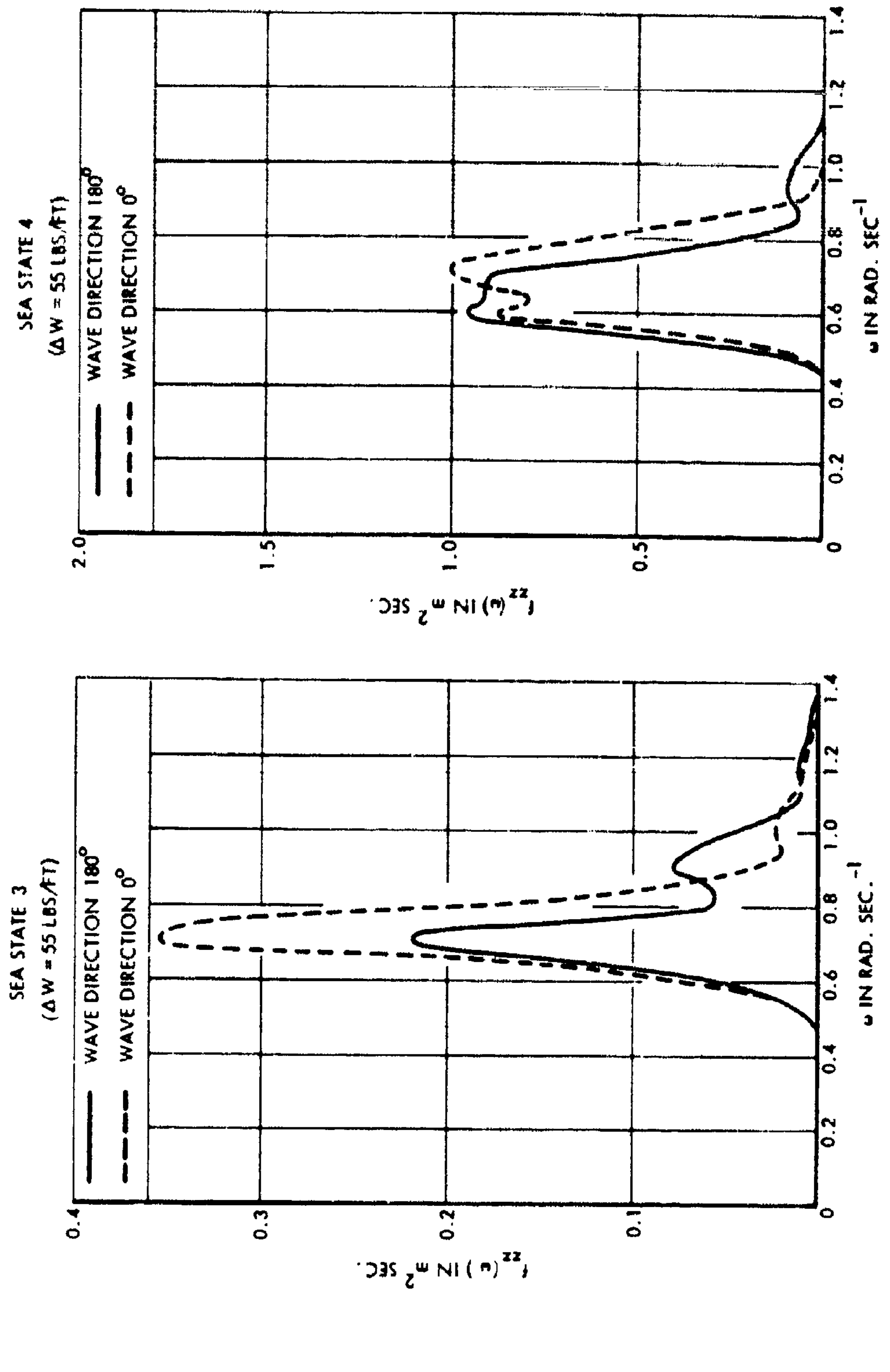


FIGURE 21 - (CONCLUDED)

RELATIVE HEAVE SPECTRA



NOTE: ΔW = UNIT WEIGHT OF MOORING LINE

FIGURE 22 - MEASURED SPECTRA OF RELATIVE HEAVE BETWEEN COMET AND PAGE IN IRREGULAR WAVE TEST

RELATIVE HEAVE SPECTRA

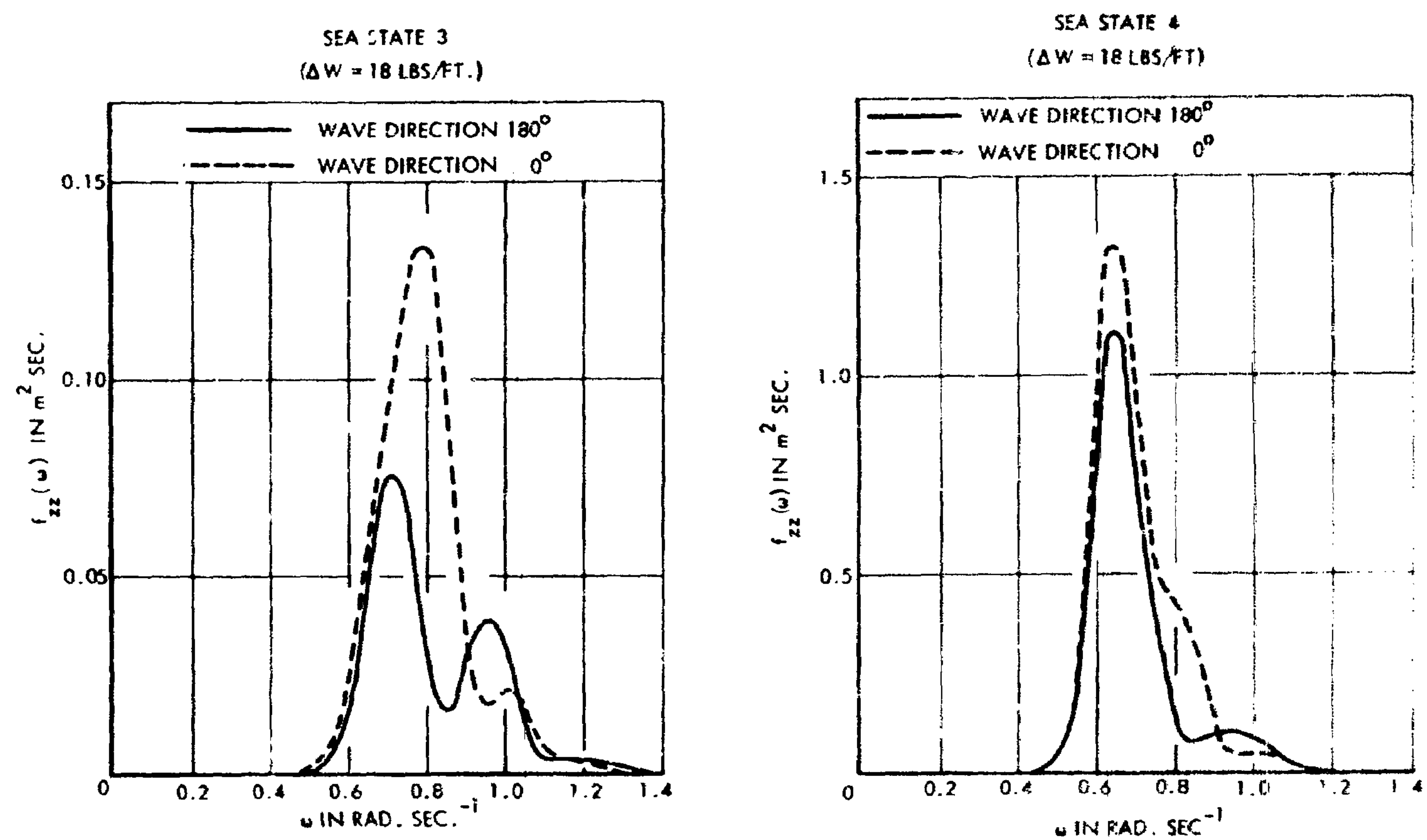


FIGURE 22 - (CONTINUED)

RELATIVE HEAVE SPECTRA

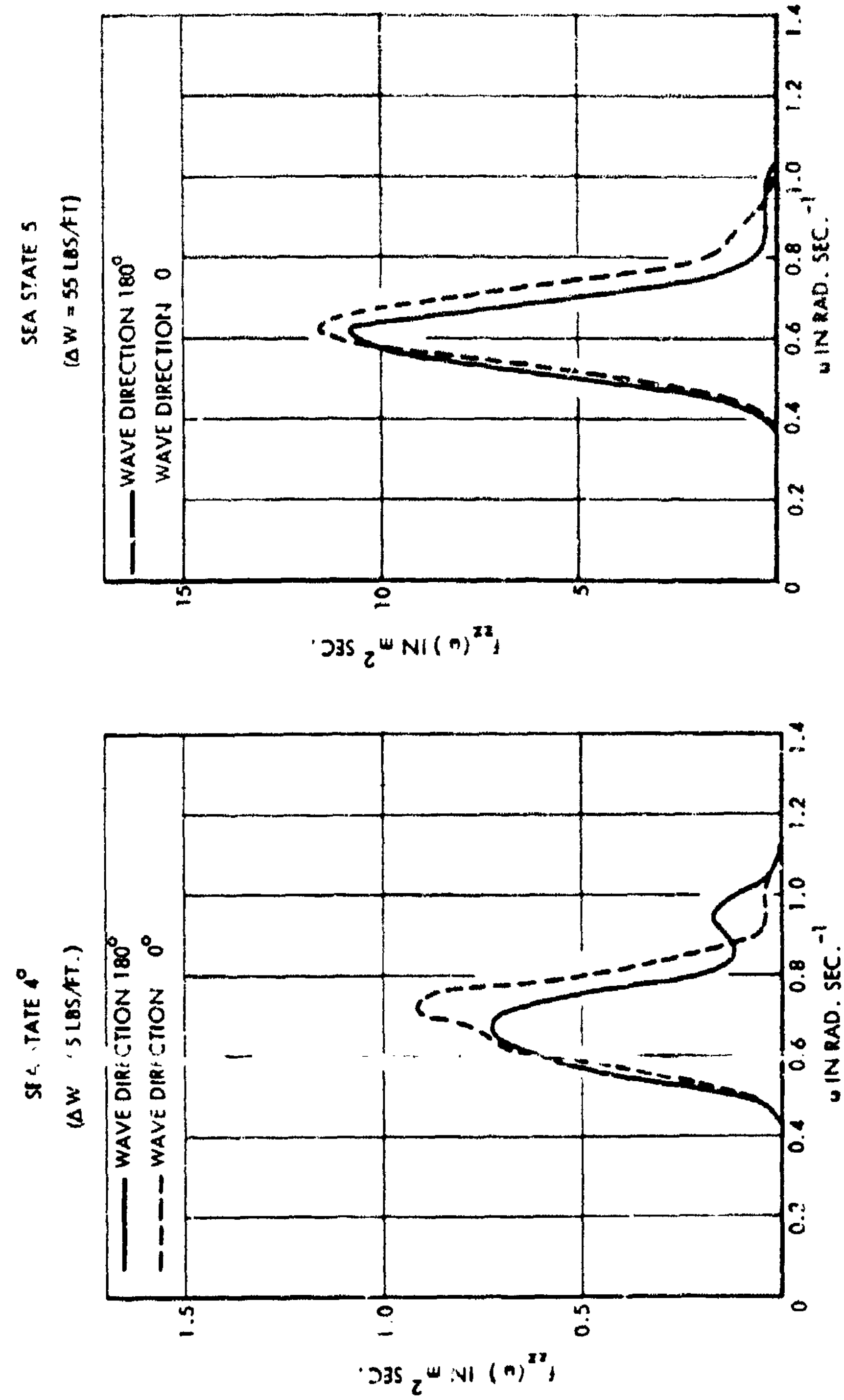


FIGURE 22 - (CONTINUED)

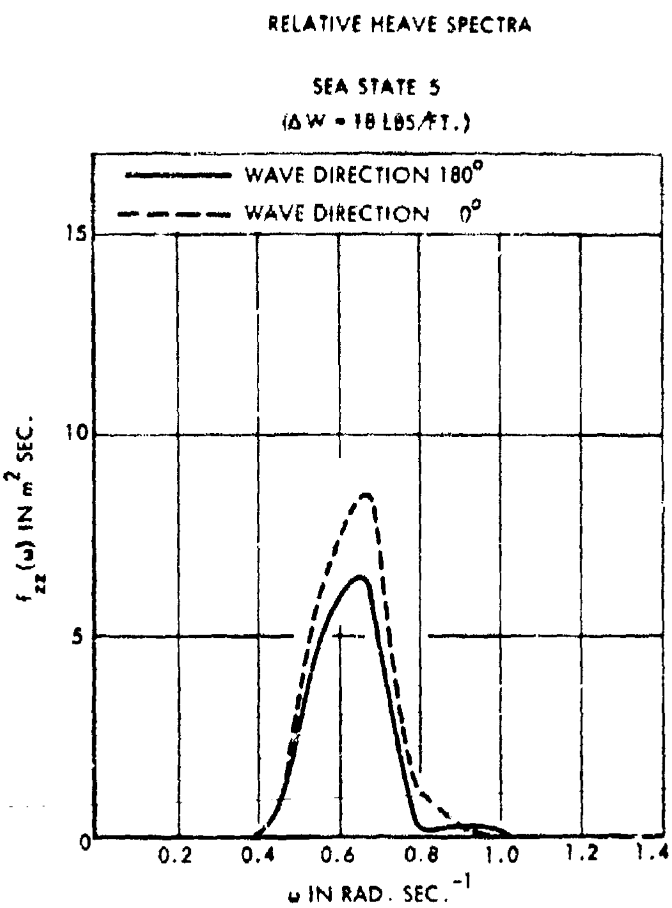
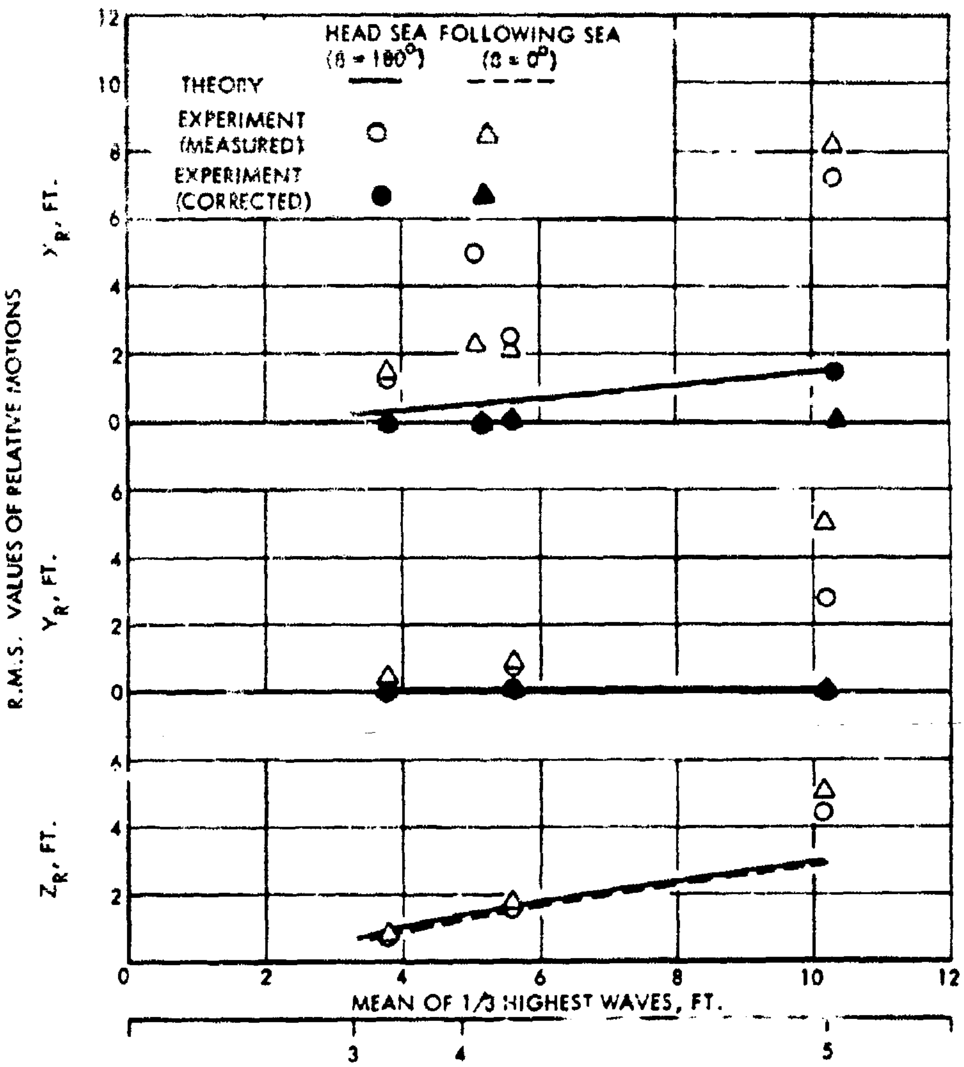
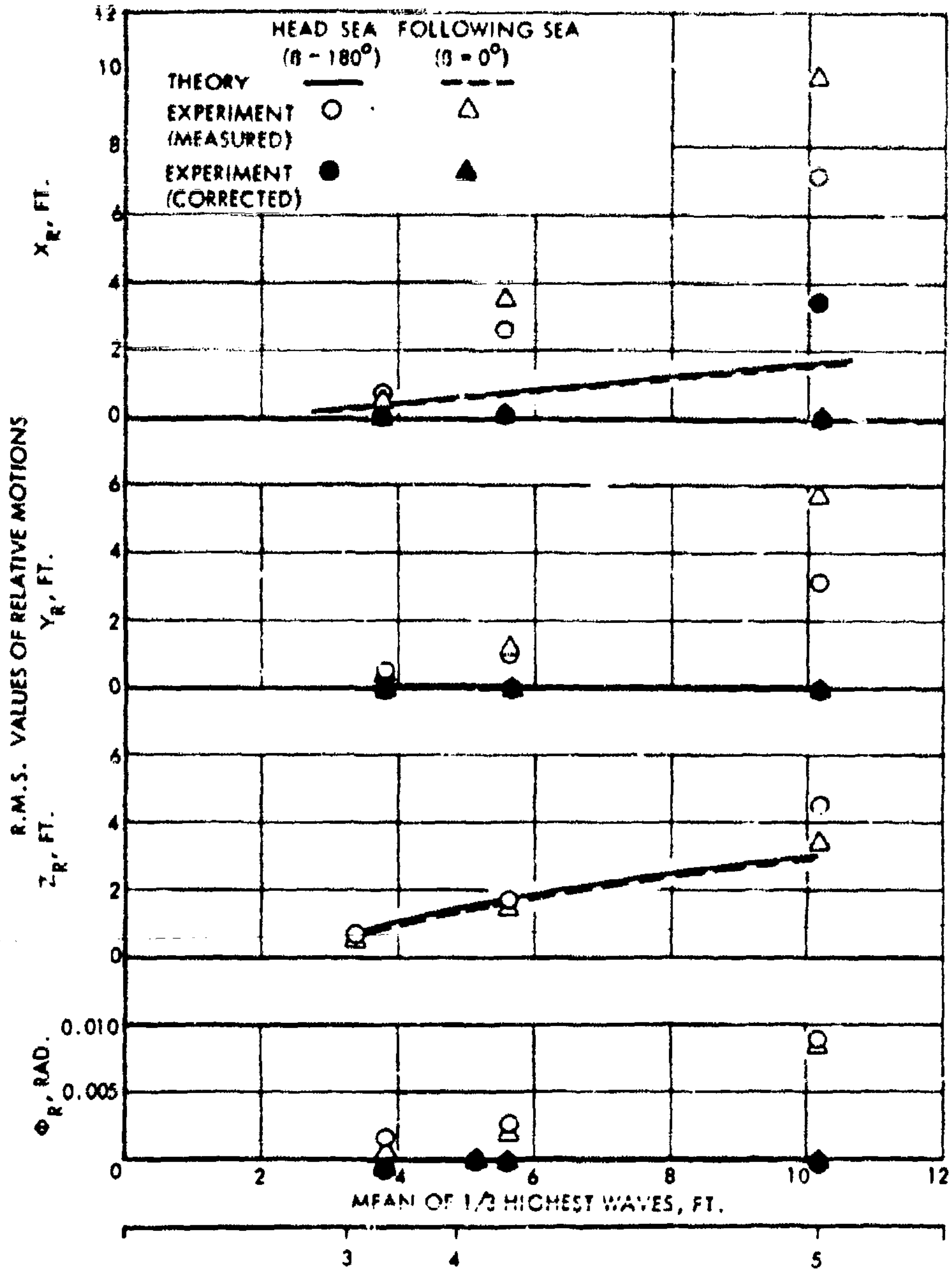


FIGURE 22 (CONCLUDED)



(a) WEIGHT OF MOORING LINE = 55 LBS/FT.

FIGURE 23 - COMPARISON BETWEEN THEORETICAL AND EXPERIMENTAL R.M.S. VALUES OF RELATIVE MOTIONS IN IRREGULAR UNIDIRECTIONAL SEA



(b) WEIGHT OF MOORING LINE = 18 LBS/FT.

FIGURE 23 - (CONCLUDED)

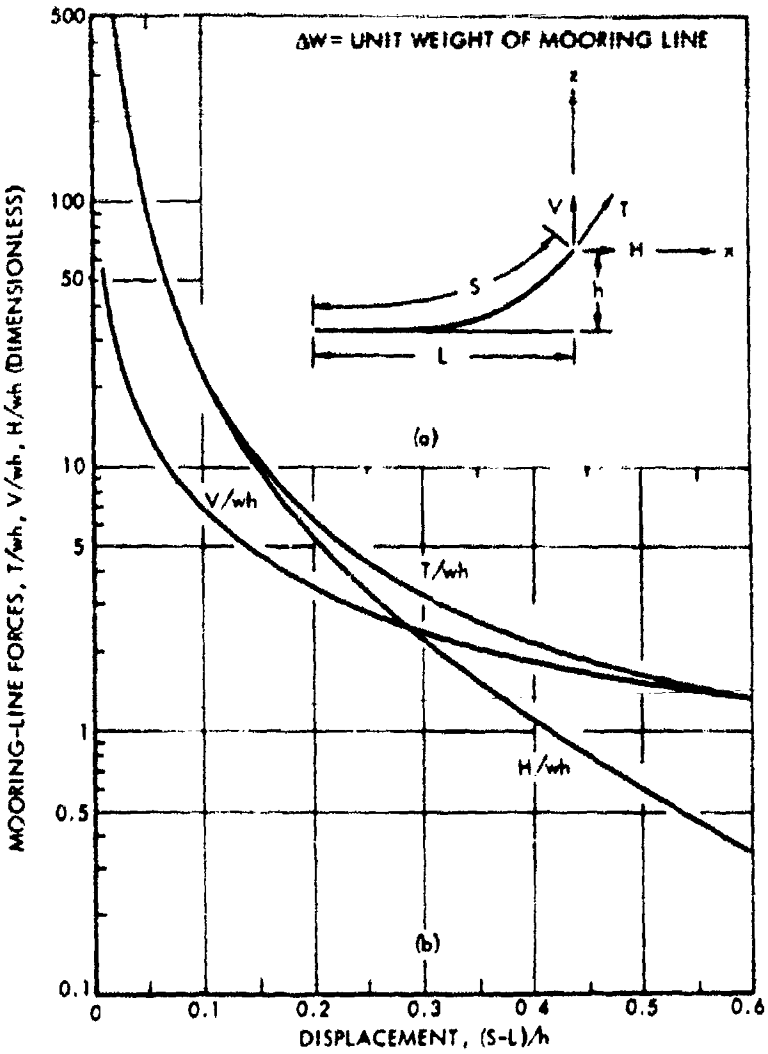


FIGURE 24 - NONDIMENSIONAL REPRESENTATION OF MOORING-LINE FORCES AS A FUNCTION OF DISPLACEMENTS

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13. ABSTRACT

The purpose of this study is to determine the optimum position for the mating of the beach discharge lighter USAV LT COL JOHN V.D. PAGE and the roll-on/roll-off ship USNS COMET. Theoretical and experimental investigations of the individual ship motions (i.e., surge, heave, sway, pitch, roll and yaw) and the relative motions between the stern of ships (i.e., relative surge, heave, sway and roll) for the above mentioned two ships were carried out at zero ship speed and various sea conditions. Results for the motions were presented as a function of wavelengths, wave directions and sea states. It was found that either head seas or following seas was the best position for the mating. Further, the shallow water effect on the ship motions were investigated and results were also presented.

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Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Ship motions Relative motion between ships Shallow water effect on ship motion Wave exciting force on ship Linearized equation of motion for ship Coefficients in the equation of motion for ship						

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